On the equivalence between the Brunn-Minkowski inequality and the CD condition

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We discuss the equivalence between the curvature dimension condition $CD(K, N)$, in the sense of Lott–Sturm–Villani, and the validity of the generalised Brunn–Minkowski inequality $BM(K, N)$. As a first step, we prove such equivalence in the setting of weighted Riemannian manifolds, where the $CD(K, N)$ condition is equivalent to a lower bound on the modified Ricci tensor. In the setting of essentially non-branching metric measure spaces, the equivalence is still an open problem. We present a preliminary result in this direction, showing that, at this level of generality, the $CD(K, N)$ condition is a equivalent to a newly introduced notion that we call strong Brunn–Minkowski inequality $SBM(K, N)$, which is a reinforcement of the generalized Brunn–Minkowski inequality $BM(K, N)$.

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