Nodal count via topological data analysis

Iosif POLTEROVICH

Université de Montréal, Canada

The celebrated Courant nodal domain theorem implies that the number of nodal domains of a Laplace eigenfunction is controlled by the corresponding eigenvalue. There have been many attempts to find an appropriate generalization of this statement in various directions: to linear combinations of eigenfunctions, to their products, to other operators. It turns out that these and other extensions of Courant's theorem can be obtained if one counts the nodal domains in a coarse way, i.e. ignoring small oscillations. The proof uses multiscale polynomial approximation in Sobolev spaces and the theory of persistence barcodes originating in topological data analysis.

The talk is based on a joint work with L. Buhovsky, J. Payette, L. Polterovich, E. Shelukhin and V. Stojisavljević.