Hierarchy structures in CMC surfaces of finite index

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Given $\varepsilon > 0$, $I \in \mathbb{N} \cup \{0\}$ and $K_0, H_0 \geq 0$, let X be a complete Riemannian 3-manifold with injectivity radius $\mathrm{Inj}(X) \geq \varepsilon$ and with the supremum of absolute sectional curvature at most K_0, and let M be a complete immersed surface in X of constant mean curvature $H \in [0, H_0]$ with Morse index at most I. For such M, we will describe how the interesting ambient geometry of the immersion is organized locally around at most I points of M where the norm of the second fundamental form takes on large local maximum values, and where the local pictures on different blowing-up scales are complete immersed minimal surfaces in $\mathbb{R}^3$ with index at most I and finitely many branch points. The novelty of our analysis consists of understanding clustering phenomena, i.e. multiple blow-up limits appearing at different scales around the same point of M.

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