From approximation to diffeomorphism. Applications

Sylvestre GALLOT

Université Grenoble-Alpes, France

It is a work in progress, in collaboration with G. Besson, G. Courtois and A. Sambusetti.

We consider two complete Riemannian manifolds X^n and Y^n such that there exists a local Gromov-Hausdorff ε-approximation $f : X^n \to Y^n$. We suppose that the curvature σ_Y of Y^n verifies $-1 \leq \sigma_Y \leq 1$; there is no curvature assumption on X^n. Using a Riemannian version of the barycentre of a measure on Y^n depending on $x \in X^n$, we slighly deform f to an explicit differentiable map $X^n \to Y^n$, which occurs to be a diffeomorphism when $0 < \varepsilon < \varepsilon_0$; we compute ε_0 in terms of n and of a lower bound of the convexity radii of X^n and Y^n. Up to now, applications are results of rigidity for differential structures (among which the canonical structures of $\mathbb{R}^4$ and $\mathbb{S}^4$) and finiteness (up to diffeomorphisms) for closed manifolds.