$$\cancel{R_i \mapsto R_i r} \quad \cancel{\rightsquigarrow}$$

$$\det \begin{pmatrix} 3 & -3 & 3 \\ 7 & -5 & 2 \\ -5 & 2 & 1 \end{pmatrix} = 3 \det \begin{pmatrix} 1 & -1 & 1 \\ 7 & -5 & 2 \\ -5 & 2 & 1 \end{pmatrix}$$

$$\det A = \sum_k (-1)^{1+k} a_{k1} \det(A_{k1})$$

$$= 3 \det \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -5 \\ 0 & -3 & 6 \end{pmatrix} =$$

$$= 3 \det \begin{pmatrix} 2 & -5 \\ -3 & 6 \end{pmatrix} = 3 \det \begin{pmatrix} -1 & 1 \\ -3 & 6 \end{pmatrix}$$

$$= 9 \det \begin{pmatrix} 1 & -1 \\ 1 & -2 \end{pmatrix} = 9 \det \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} = -9.$$

$$\det \begin{pmatrix} 1 & 4 & 4 & 3 & 3 \\ 3 & 1 & 3 & 2 & 1 \\ 3 & 1 & 5 & 1 & 2 \\ 4 & 2 & 4 & 4 & 1 \\ 4 & 3 & 5 & 4 & 2 \end{pmatrix} = \det \begin{pmatrix} 1 & 4 & 4 & 3 & 3 \\ 0 & -11 & -9 & -7 & -8 \\ 0 & -11 & -7 & -8 & -7 \\ 0 & -14 & -12 & -8 & -11 \\ 0 & -13 & -11 & -8 & -10 \end{pmatrix}$$

$$= \det \begin{pmatrix} -11 & -9 & -7 & -8 \\ -11 & -7 & -8 & -7 \\ -14 & -12 & -8 & -11 \\ -13 & -11 & -8 & -10 \end{pmatrix}$$

$$= \det \begin{pmatrix} -11 & -9 & -7 & -8 \\ -11 & -7 & -8 & -7 \\ -14 & -12 & -8 & -11 \\ -13 & -11 & -8 & -10 \end{pmatrix}$$

$$= \det \begin{pmatrix} -11 & -9 & -7 & -8 \\ 0 & 2 & 1 & 1 \\ -3 & -3 & -1 & -3 \\ -2 & -2 & -1 & -2 \end{pmatrix}$$

$$= \det \begin{pmatrix} -11 & -9 & -7 & -8 \\ 0 & 2 & 1 & 1 \\ -3 & -3 & -1 & -3 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$= \det \begin{pmatrix} 0 & 2 & -7 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$= \det \begin{pmatrix} 0 & 2 & -7 & 3 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}$$

$$d^{(n)} = \sum_k (-1)^{k+1} a_{k1} d^{(n-1)}(A_{k1})$$

$$\overset{\downarrow}{=}$$

$$= - \det \begin{pmatrix} 2 & -7 & 3 \\ 2 & 1 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= - \det \begin{pmatrix} 2 & -7 & 3 \\ 0 & 8 & -2 \\ 0 & -1 & 0 \end{pmatrix}$$

$$= -2 \det \begin{pmatrix} 8 & -2 \\ -1 & 0 \end{pmatrix}$$

$$= -2 \left(0 - (-1)(-2)\right) = 4$$

$$\det\begin{pmatrix} 13247 & 28469 \\ 13347 & 28569 \end{pmatrix} =$$

$$= \det\begin{pmatrix} 13247 & 28469 \\ 100 & 100 \end{pmatrix}$$

$$= 100 \cdot \det\begin{pmatrix} 13247 & 28469 \\ 1 & 1 \end{pmatrix}$$

$$= 100 \det\begin{pmatrix} 0 & 15222 \\ 1 & 1 \end{pmatrix} =$$

$$= -100 \det\begin{pmatrix} 1 & 1 \\ 0 & 15222 \end{pmatrix}$$

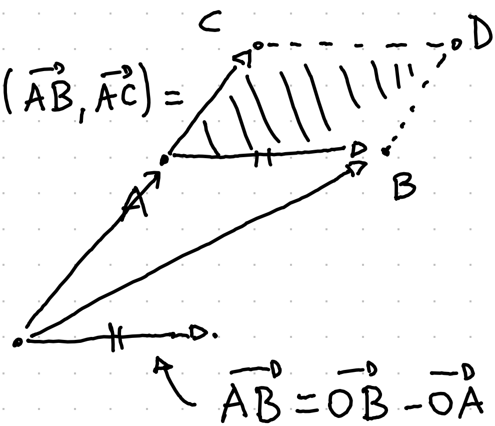
$$= -100 \cdot 15222.$$

det 2x2 come area orientata

$A, B, C \in \mathcal{E}^2$

$$\text{Area}\left(\mathcal{P}(\vec{AB}, \vec{AC})\right) = \left|\det\left(F_{\mathcal{B}}(\vec{AB})^t, F_{\mathcal{B}}(\vec{AC})^t\right)\right|$$

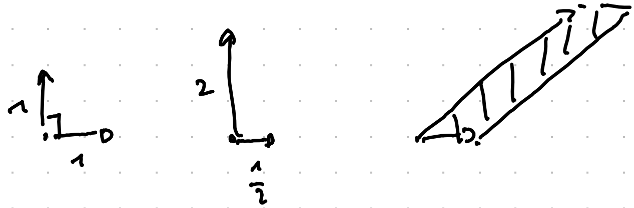
$\mathcal{P}(\vec{AB}, \vec{AC}) =$

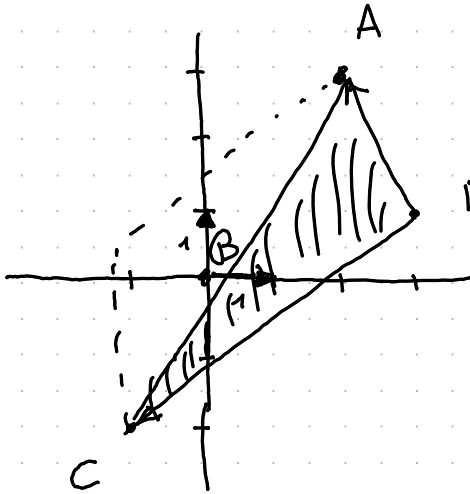


$\vec{AB} = \vec{OB} - \vec{OA}$

$\vec{AD} = \vec{AB} + \vec{AC}$

$\vec{AB} = \vec{OB} - \vec{OA}$.





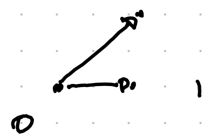
$F_{\mathcal{B}}(\overrightarrow{OA})$

$A \overset{\downarrow}{=} \begin{pmatrix}2\\3\end{pmatrix}$

$B = \begin{pmatrix}3\\1\end{pmatrix}$

$C = \begin{pmatrix}-1\\-2\end{pmatrix}$

A

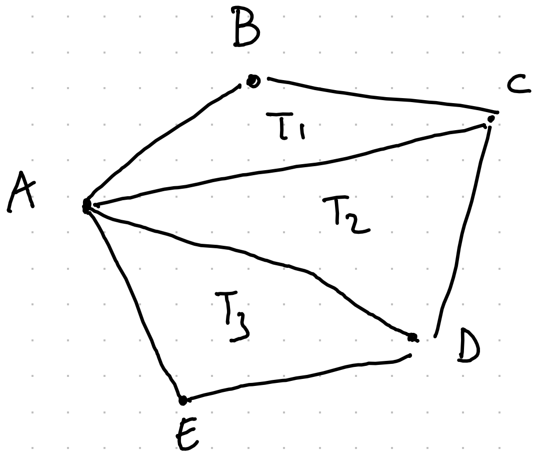


$$\text{Area}(\overset{\triangle}{ABC}) = \frac{1}{2}\,\text{Area}\left(\mathcal{P}(\overrightarrow{BA}, \overrightarrow{BC})\right)$$

$$= \frac{1}{2}\left|\det\left(F_{\mathcal{B}}(\overrightarrow{BA})^t, F_{\mathcal{B}}(\overrightarrow{BC})^t\right)\right|$$

$$= \frac{1}{2}\left|\det\left(\begin{pmatrix}2\\3\end{pmatrix}^t - \begin{pmatrix}3\\1\end{pmatrix}^t, \begin{pmatrix}-1\\-2\end{pmatrix}^t - \begin{pmatrix}3\\1\end{pmatrix}^t\right)\right|$$

$$= \frac{1}{2}\left|\det\begin{pmatrix}-1 & 2\\-4 & -3\end{pmatrix}\right| = \frac{1}{2}|3+8| = \frac{11}{2}.$$



$$\text{Area} = \text{Area}(T_1) + \text{Area}(T_2) + \text{Area}(T_3)$$

$$= \frac{1}{2}\Big(|\det(B-A, C-A)| + |\det(C-A, D-A)| + |\det(D-A, E-A)| \Big).$$

$$B - A := \vec{OB} - \vec{OA}$$

$$\det \begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix} = \det \big((3,3), (2,4) \big)$$

$$= \det \big(3\,(1,1), (2,4) \big)$$

$$= \det \left(D_1(3) \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix} \right)$$

$$= 3 \det \begin{pmatrix} 1 & 1 \\ 2 & 4 \end{pmatrix}$$

$A \in Mat_{n\times n}(\mathbb{K})$.

$\det A \neq 0 \iff rg\, A = n$

Infatti:

$$A \underset{R}{\rightsquigarrow} rref\, A = R$$

2 possibilità

① R ha l'ultima riga nulla

② $R = \mathbb{1}_n$.

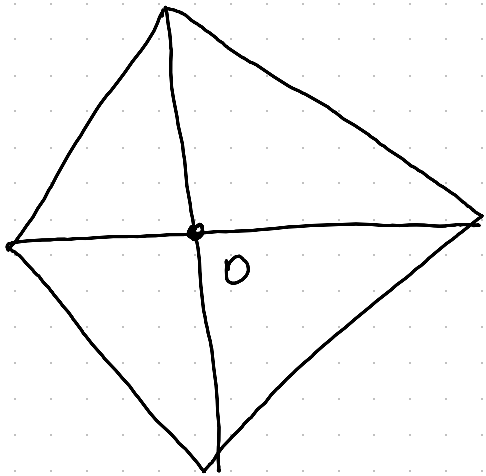
① $\iff rg\, R < n \iff rg\, A < n$.

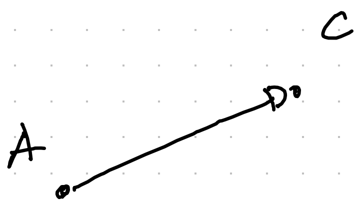
$\det A = c \det(R)$

dove $c^{\neq 0}$ è una costante che dipende solo dalle operazioni elementari utilizzate per trasformare A in R.

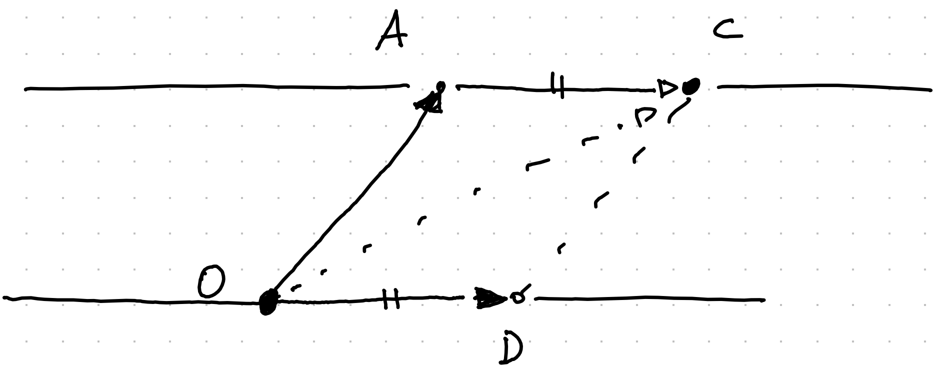
Nel caso 1: $\det A = 0$

Nel caso 2: $\det A = c \neq 0$.





$$\boxed{\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}}$$



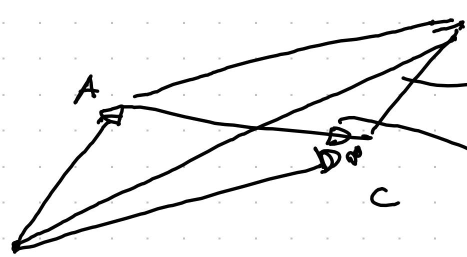
$\vec{AC} \equiv \vec{OD}$

·) $|\overline{OD}| = |\overline{AC}|$

·) stesso verso ~~orientazione~~

·) stessa direzione

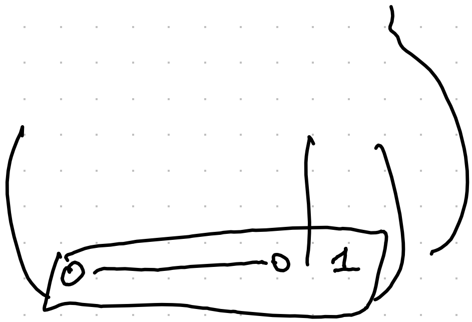
$$\vec{OC} = \vec{OA} + \vec{OD} \Rightarrow \vec{AC} = \vec{OD} = \vec{OC} - \vec{OA}$$



somme.

differenza

$$0x = 1$$



$$\det\begin{pmatrix} 0 & x_1 & x_2 & x_3 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 0 & 1 \end{pmatrix} = \det\begin{pmatrix} -x_1^2 & 0 & x_2 & x_3 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 0 & 1 \end{pmatrix}$$

$$= \det\begin{pmatrix} -x_1^2-x_2^2 & 0 & 0 & x_3 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 0 & 1 \end{pmatrix}$$

$$= \det\begin{pmatrix} -x_1^2-x_2^2-x_3^2 & 0 & 0 & 0 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 0 & 1 \end{pmatrix}$$

$$= \det \begin{pmatrix} -x_1^2-x_2^2-x_3^2 & 0 & 0 & 0 \\ x_1 & 1 & 0 & 0 \\ x_2 & 0 & 1 & 0 \\ x_3 & 0 & 0 & 1 \end{pmatrix}$$

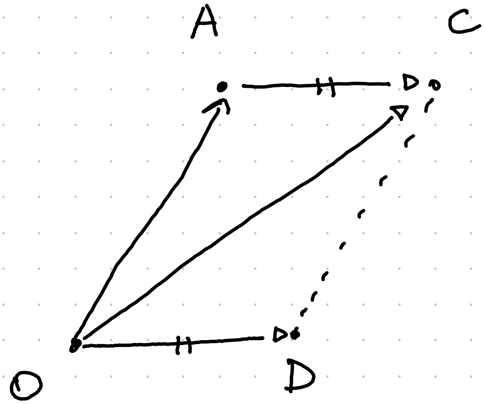
$$= -\left(x_1^2+x_2^2+x_3^2\right) \det \mathbb{1}_3 +$$

$$- x_1 \det \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} +$$

$$+ x_2 \det \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$- x_3 \det \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = -\left(x_1^2+x_2^2+x_3^2\right)$$

$$195 = 100 + 90 + 5$$



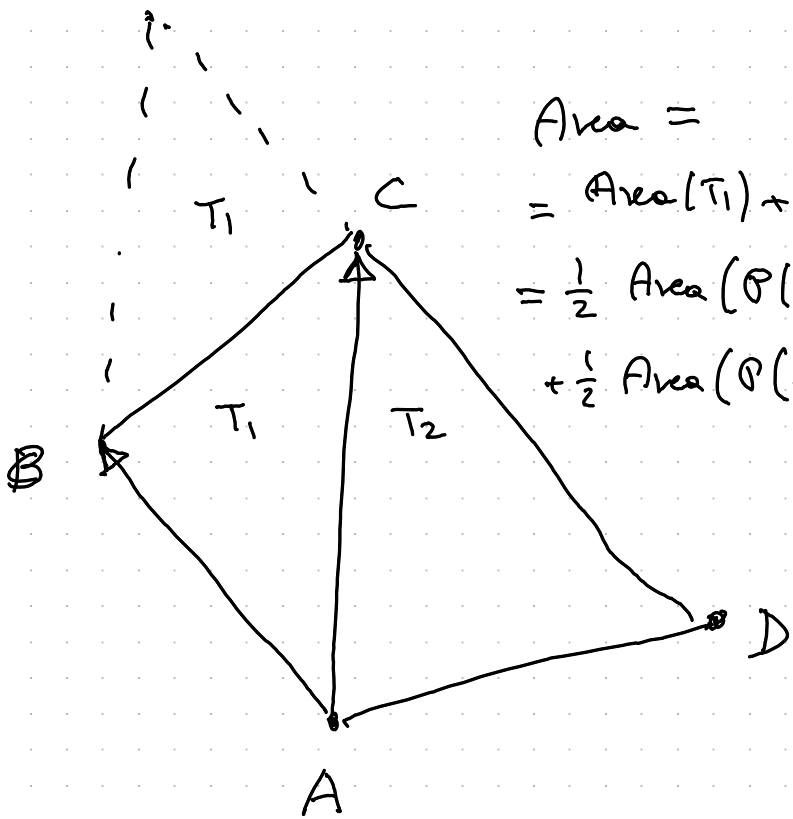
$$\boxed{\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}}$$

$\overrightarrow{AC} \equiv \overrightarrow{OD}$

1) $|\overline{OD}| = |\overline{AC}|$
2) stessa direzione
3) stesso verso.

$$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{OA}$$

$$\overrightarrow{AC} = \overrightarrow{OD} = \overrightarrow{OC} - \overrightarrow{OA}$$



Area =

$= \text{Area}(T_1) + \text{Area}(T_2)$

$= \frac{1}{2}\,\text{Area}(P(\vec{AB}, \vec{AC})) +$

$+ \frac{1}{2}\,\text{Area}(P(\vec{AC}, \vec{AD}))$

$$\text{Area } P(\vec{AC}, \vec{AB}) = \left|\det\left(F_B(\vec{AC})^t, F_B(\vec{AB})^t\right)\right|$$

$$= \left|\det\left(F_B(\vec{OC} - \vec{OA}), F_B(\vec{OB} - \vec{OA})\right)\right|$$

$$= \left|\det\left(F_B(\vec{OC}) - F_B(\vec{OA}), F_B(\vec{OB}) - F_B(\vec{OA})\right)\right|$$

$$\det\begin{pmatrix} -2 & -4 & -2 & 4 \\ -3 & 4 & 1 & 2 \\ -3 & 3 & -4 & -1 \\ -1 & 3 & -1 & 1 \end{pmatrix}$$

$$= \det\begin{pmatrix} -2 & -4 & -2 & 4 \\ 0 & 1 & 5 & 3 \\ -3 & 3 & -4 & -1 \\ -1 & 3 & -1 & 1 \end{pmatrix}$$

$$= \det\begin{pmatrix} 1 & -7 & 2 & 5 \\ 0 & 1 & 5 & 3 \\ -3 & 3 & -4 & -1 \\ -1 & 3 & -1 & 1 \end{pmatrix}$$

$$= \det\begin{pmatrix} 1 & -7 & 2 & 5 \\ 0 & 1 & 5 & 3 \\ 0 & -18 & 2 & 14 \\ 0 & -4 & 1 & 6 \end{pmatrix}$$

$$= \det\begin{pmatrix} 1 & 5 & 3 \\ -18 & 2 & 14 \\ -4 & 1 & 6 \end{pmatrix} = 2\det\begin{pmatrix} 1 & 5 & 3 \\ -9 & 1 & 7 \\ -4 & 1 & 6 \end{pmatrix}$$

$$2 \det \begin{pmatrix} 1 & 5 & 3 \\ -9 & 1 & 7 \\ -4 & 1 & 6 \end{pmatrix}$$

$$= 2 \det \begin{pmatrix} 1 & 5 & 3 \\ 0 & 46 & 34 \\ 0 & 21 & 18 \end{pmatrix}$$

$$= 2 \det \begin{pmatrix} 1 & 5 & 3 \\ 0 & 4 & -2 \\ 0 & 21 & 18 \end{pmatrix}$$

$$= 2 \cdot 2 \cdot 3 \det \begin{pmatrix} 1 & 5 & 3 \\ 0 & 2 & -1 \\ 0 & 7 & 6 \end{pmatrix}$$

$$= 2 \cdot 2 \cdot 3 \det \begin{pmatrix} 2 & -1 \\ 7 & 6 \end{pmatrix}$$

$$= 2 \cdot 2 \cdot 3 \det \begin{pmatrix} 2 & -1 \\ 1 & 9 \end{pmatrix}$$

$$= -2 \cdot 2 \cdot 3 \det \begin{pmatrix} 1 & 9 \\ 2 & -1 \end{pmatrix} = -2 \cdot 2 \cdot 3 \det \begin{pmatrix} 1 & 9 \\ 0 & -19 \end{pmatrix}$$

$$= 2 \cdot 2 \cdot 3 \cdot 19.$$

$$\det \begin{pmatrix} 2 \cdot a & 2b \\ 2c & 2d \end{pmatrix} = 2 \det \begin{pmatrix} a & b \\ 2c & 2d \end{pmatrix}$$

$$= 2^2 \det \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\det \lambda A = \lambda^n \det A.$$

$$\det \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \det \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix} = -2$$

$$\det \begin{pmatrix} \lambda & 2\lambda \\ 3\lambda & 4\lambda \end{pmatrix} = \det \begin{pmatrix} \lambda & 2\lambda \\ 0 & -2\lambda \end{pmatrix} = -2\lambda^2$$

$V \supset \mathcal{B}$ base.

$$\det\left(F_{\mathcal{B}}(w_1)^t, \dots, F_{\mathcal{B}}(w_n)^t\right) \neq 0$$

$\Longleftrightarrow$ $(w_1, \dots, w_n)$ è una base di V.

$$(x+\lambda_1)^2 = x^2 + 2\lambda_1 + \lambda_1^2$$

$$(x+\lambda_2)^2 = x^2 + 2\lambda_2 + \lambda_2^2$$

$$(x+\lambda_3)^2 = x^2 + 2\lambda_3 + \lambda_3^2$$

$$\det\begin{pmatrix} 1 & 2\lambda_1 & \lambda_1^2 \\ 1 & 2\lambda_2 & \lambda_2^2 \\ 1 & 2\lambda_3 & \lambda_3^2 \end{pmatrix} = \dots = 2(\lambda_2-\lambda_1)(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)$$