$$\mathrm{Ker}\,(a,b) = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid ax+by=0 \right\}$$

$(a,b)\neq(0,0)$. $\quad = \left\langle \begin{pmatrix} -b \\ a \end{pmatrix} \right\rangle$

Es 1: $V \xrightarrow{\mathcal{L}} V$ lineare.

$\mathcal{B}_{\mathrm{Ker}\,\mathcal{L}} = (v_1, \dots, v_{n-r})$ $\rightarrow$ Estendiamola ad una base di V $\quad \mathcal{B}_1 = (v_1, \dots, v_{n-r}, v_{n-r+1}, \dots, v_n)$ $\quad r = \mathrm{rg}(\mathcal{L})$. $n = \dim V$.

$$\mathcal{B}_{\mathrm{Im}\,\mathcal{L}} = (\underbrace{\mathcal{L}(v_{n-r+1})}_{= w_1}, \dots, \underbrace{\mathcal{L}(v_n)}_{= w_r})$$

Formula della dimensione

Estendiamola ad una base di V

$$\mathcal{B}_2 = (w_1, \dots, w_r, w_{r+1}, \dots, w_n)$$

$$\mathcal{L}(v_1) = 0_V = \mathcal{L}(v_2) = \dots = \mathcal{L}(v_{n-r})$$

$$\mathcal{L}(v_{n-r+1}) = w_1,\ \mathcal{L}(v_{n-r+2}) = w_2, \dots, \mathcal{L}(v_n) = w_r.$$

$$\begin{array}{ccc} V & \xrightarrow{\mathcal{L}} & V \\ \downarrow F_{\mathcal{B}_1} & & \downarrow F_{\mathcal{B}_2} \\ \mathbb{K}^n & \xrightarrow{S_D} & \mathbb{K}^n \end{array}$$

$$D = \begin{array}{c} \quad\ \ 1 \qquad\quad\ n-r \\ \begin{pmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ & & & 0 & 1 & \cdots & 0 \\ & & & \vdots & & & \\ & & & 0 & \cdots & & 1 \\ 0 & & 0 & 0 & \cdots & & 0 \\ 0 & \cdots & 0 & 0 & \cdots & & 0 \end{pmatrix} \end{array}$$

Riordiniamo $\mathcal{B}_1$: $\rightarrow$ $D = \left(\begin{array}{c|c} \mathbb{1}_r & 0 \\ \hline 0 & 0 \end{array}\right)$

$$\mathcal{B}_1 = (v_{n-r+1}, v_{n-r+2}, \dots, v_n, v_1, v_2, \dots, v_{n-r})$$

Es: $\mathcal{L}: \mathbb{R}[x]_{\leq 2} \to \mathbb{R}[x]_{\leq 2}$

$$\mathcal{L}(p) = p(x+1) - p'(x^2)$$

$\mathcal{B}_{\mathrm{Ker}\,\mathcal{L}} = ?$

$$\mathcal{L}(1) = 1$$
$$\mathcal{L}(x) = x+1-1 = x$$
$$\mathcal{L}(x^2) = (x+1)^2 - 2(x^2) =$$
$$= x^2+2x+1-2x^2 = -x^2+2x+1$$

$$\begin{tikzcd} \mathbb{R}[x]_{\leq 2} \arrow[r, "\mathcal{L}"] \arrow[d, "F_e"'] & \mathbb{R}[x]_{\leq 2} \arrow[d, "F_e"] \\ \mathbb{R}^3 \arrow[r] & \mathbb{R}^3 \end{tikzcd}$$

$e = (1, x, x^2)$

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$$

$\mathrm{Ker}\,A = \{0_{\mathbb{R}^3}\} \Rightarrow \mathrm{Ker}\,\mathcal{L} = \{0_{\mathbb{R}[x]_{\leq 2}}\}$

$\mathcal{B}_{\mathrm{Ker}\,\mathcal{L}} = \emptyset.$

$\mathcal{B}_1 = e = (1, x, x^2)$

$\mathcal{B}_2 = \mathcal{L}(\mathcal{B}_1) = (1, x, -x^2+2x+1)$

$$\begin{tikzcd} \cdot \arrow[r, "\mathcal{L}"] \arrow[d, "F_{\mathcal{B}_1}"'] & \cdot \arrow[d, "F_{\mathcal{B}_2}"] \\ \cdot \arrow[r, "D"] & \cdot \end{tikzcd}$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\mathcal{L}$ è diagonalizzabile $\Leftrightarrow$ A è diagonalizzabile

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix}$$

$Sp(A) = \{1, -1\}$. $\quad ma_A(1) = 2 \quad ma_A(-1) = 1$.

$\Rightarrow mg_A(-1) = 1$.

$$mg_A(1) = \dim \operatorname{Ker} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{pmatrix}$$

$$= \dim \operatorname{Ker} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = 2$$

$\Rightarrow A$ è diagonalizzabile.

Base di autovettori:

$x_3 = 0$

$$V_1(A) = \operatorname{Ker} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle$$

$$V_{-1}(A) = \operatorname{Ker} \begin{pmatrix} -2 & 0 & -1 \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{pmatrix} = \operatorname{Ker} \begin{pmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1 \end{pmatrix}$$

$$= \left\langle \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} \right\rangle$$

Verifichiamo

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = - \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}$$

$\mathcal{L}: V \to V$ diagonalizzabile

$\iff$ A è diagonalizzabile per ogni A t.c.

$$\begin{tikzcd} V \arrow[r, "\mathcal{L}"] \arrow[d, "F_B"'] & V \arrow[d, "F_B"] \\ \mathbb{K}^n \arrow[r, "S_A"] & \mathbb{K}^n \end{tikzcd}$$

Sol.: Se $\mathcal{L}$ è diagonalizzabile allora $\exists$ $B \subset V$ base t.c.

$$\begin{tikzcd} V \arrow[r, "\mathcal{L}"] \arrow[d, "F_B"] & V \arrow[d, "F_B"] \\ \mathbb{K}^n \arrow[r, "S_D"] & \mathbb{K}^n \end{tikzcd}$$

con $D = \mathrm{diag}(\lambda_1, \dots, \lambda_n)$

Sia $B' \subset V$ un'altra base e A la matrice che rappresenta $\mathcal{L}$ nella base B' (sia in partenza che in arrivo). Dimostriamo che A è diagonalizzabile.

$$\begin{tikzcd}
V \arrow[r, equal] \arrow[d, "F_B"'] & V \arrow[r, "\mathcal{L}"] \arrow[d, "F_{B'}"] & V \arrow[r, equal] \arrow[d, "F_{B'}"] & V \arrow[d, "F_B"] \\
\mathbb{K}^n \arrow[r, "S_C"] \arrow[rrr, bend right, "S_D"'] & \mathbb{K}^n \arrow[r, "S_A"] & \mathbb{K}^n & \mathbb{K}^n \arrow[l, "S_C"']
\end{tikzcd}$$

$\Rightarrow \quad C^{-1}AC = D \quad \Rightarrow A$ è diagonalizzabile.

Viceversa, sia $\mathcal{B} \subset V$ base di V t.c. la matrice A che rappresenta $\mathcal{L}$ in questa base è diagonalizzabile. Dimostriamo che $\mathcal{L}$ è diagonalizzabile.

Per ipotesi: $\exists\, C$ invertibile e D diagonale t.c.

$$C^{-1}AC = D. \quad (*)$$

Sia $\mathcal{C} = (C^1, \dots, C^n) \subset \mathbb{K}^n$ è una base.

$$\begin{tikzcd}
\mathbb{K}^n \arrow[r, "S_A"] \arrow[d, "S_{C^{-1}} = F_{\mathcal{C}}"'] & \mathbb{K}^n \arrow[d, "F_{\mathcal{C}} = S_{C^{-1}}"] \\
\mathbb{K}^n \arrow[r, "S_D"] & \mathbb{K}^n
\end{tikzcd}$$

$$\begin{tikzcd}
V \arrow[r, "L"] \arrow[d, "F_{\mathcal{B}}"'] \arrow[dd, bend right=60, "F_e \circ F_{\mathcal{B}}"'] & V \arrow[d, "F_{\mathcal{B}}"] \arrow[dd, bend left=60, "F_e \circ F_{\mathcal{B}}"] \\
\mathbb{K}^n \arrow[r, "S_A"] \arrow[d, "F_e"'] & \mathbb{K}^n \arrow[d, "F_e"] \\
\mathbb{K}^n \arrow[r, "S_D"] & \mathbb{K}^n
\end{tikzcd}$$

$F_e \circ F_{\mathcal{B}}$ è un isom.

$F_e \circ F_{\mathcal{B}}$ è un isom.

$F_e \circ F_{\mathcal{B}} = F_{\mathcal{B}'}$ dove

$\mathcal{B}' = (F_e \circ F_{\mathcal{B}})^{-1}(e_1, \dots, e_n)$. ∎

è una base di V.

Es: Se $C = B^{-1} A B$.

$$\begin{tikzcd} \mathbb{K}^n \arrow[r, "S_A"] \arrow[d, "\simeq"', "F_{\mathcal{B}} = S_{B^{-1}}"] & \mathbb{K}^n \arrow[d, "\simeq", "S_{B^{-1}} = F_{\mathcal{B}}"'] \\ \mathbb{K}^n \arrow[r, "S_C"] & \mathbb{K}^n \end{tikzcd}$$

$\mathcal{B} = (B^1, \dots, B^n) \subset \mathbb{K}^n$ base di $\mathbb{K}^n$

$P_A(x) = P_C(x) \Rightarrow Sp(A) = Sp(C)$.

Sia $\lambda \in Sp(A) = Sp(C)$, $\lambda \in \mathbb{K}$.

$V_\lambda(A) = \{ X \in \mathbb{K}^n \mid AX = \lambda X \} = Ker(\lambda \mathbb{1}_n - A)$.

Dimostriamo che

$\boxed{F_{\mathcal{B}}(V_\lambda(A)) = V_\lambda(C).} \Rightarrow \dim V_\lambda(A) = \dim V_\lambda(C)$

Infatti:

$$F_{\mathcal{B}}(V_\lambda(A)) = \{ F_{\mathcal{B}}(X) \mid AX = \lambda X \}.$$

$$= \{ B^{-1} X \mid AX = \lambda X \}.$$

$$= \{ Y \mid ABY = \lambda BY \}. \ni Y$$

$Y = B^{-1} X$

$X = BY$ $\quad CY = B^{-1}ABY = B^{-1}(\lambda BY) = \lambda Y \Rightarrow Y \in V_\lambda(C)$

Es 5:

$$\begin{tikzcd} V \arrow[r, "L"] \arrow[d, "F_e"'] & V \arrow[d, "F_e"] \\ \mathbb{R}^3 \arrow[r, "S_A"] & \mathbb{R}^3 \end{tikzcd}$$

$L(1) = 0$

$L(x) = 0$

$L(x^2) = 2(x^2+1) - 2x \;\; = 2x^2 - 2x + 2$

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{pmatrix}$$

L è diagonalizzabile $\Leftrightarrow$ A è diag. su $\mathbb{R}$.

$Sp(A) = \{0, 2\}$. $ma_A(0) = 2$, $ma_A(2) = 1$.

$\Rightarrow$ $mg_A(1) = 1$.

$mg_A(0) = 2$ perché $Ae_1 = Ae_2 = 0_{\mathbb{R}^3}$.

$V_0(A) = \ker A = \langle e_1, e_2 \rangle$.

$x_1 = x_3$
$x_2 = -x_3$

$$V_2(A) = \ker \begin{pmatrix} 2 & 0 & -2 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} = \ker \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} = \left\langle \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\rangle$$

$\mathcal{B} = (1, x, x^2 - x + 1)$ è una base di autovettori per L.

Es 4:

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$P_A(x) = x^2 \left(x^2 - x - 3\right)$$

Sia $\lambda : \lambda^2 - \lambda - 3 = 0 \qquad \lambda^2 = \lambda + 3.$

$$V_\lambda(A) = \operatorname{Ker}\left(\lambda \mathbb{1}_4 - A\right) =$$

$$= \operatorname{Ker} \begin{pmatrix} \lambda & 0 & -1 & 0 \\ 0 & \lambda & -1 & 0 \\ -1 & -1 & \lambda-1 & -1 \\ 0 & 0 & -1 & \lambda \end{pmatrix}$$

$$= \operatorname{Ker} \begin{pmatrix} 1 & 1 & 1-\lambda & 1 \\ 0 & \lambda & -1 & 0 \\ \lambda & 0 & -1 & 0 \\ 0 & 0 & -1 & \lambda \end{pmatrix}$$

$$= \operatorname{Ker} \begin{pmatrix} 1 & 1 & 1-\lambda & 1 \\ 0 & \lambda & -1 & 0 \\ 0 & -\lambda & -1-\lambda+\lambda^2 & -\lambda \\ 0 & 0 & -1 & \lambda \end{pmatrix}$$

$$= \operatorname{Ker} \begin{pmatrix} 1 & 1 & 1-\lambda & 1 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & -2-\lambda+\lambda^2 & -\lambda \\ 0 & 0 & -1 & \lambda \end{pmatrix} = \operatorname{Ker} \begin{pmatrix} 1 & 1 & 1-\lambda & 1 \\ 0 & \lambda & -1 & 0 \\ 0 & 0 & 1 & -\lambda \\ 0 & 0 & -1 & \lambda \end{pmatrix}$$

$$\overset{\lambda \neq 0}{=} \operatorname{Ker} \begin{pmatrix} 1 & 1 & 1-\lambda & 1 \\ 0 & 1 & -1/\lambda & 0 \\ 0 & 0 & 1 & -\lambda \end{pmatrix}$$

$$= \operatorname{Ker} \begin{pmatrix} 1 & 1 & 0 & \overbrace{1+(1-\lambda)\lambda}^{-2} \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\lambda \end{pmatrix}$$

$$= \operatorname{Ker} \begin{pmatrix} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\lambda \end{pmatrix}$$

$$= \operatorname{Ker} \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -\lambda \end{pmatrix}$$

$$= \left\langle \begin{pmatrix} 1 \\ 1 \\ \lambda \\ 1 \end{pmatrix} \right\rangle$$

Es 3:

$$A = \begin{pmatrix} 4 & -2 & -4 & 6 \\ 4 & -2 & 0 & 2 \\ -4 & 6 & 4 & -2 \\ -4 & 6 & 8 & -6 \end{pmatrix}$$

$$P_A(x) = \det\left(x\mathbb{I}_4 - A\right) =$$

$$= \det\begin{pmatrix} x-4 & 2 & 4 & -6 \\ -4 & x+2 & 0 & -2 \\ 4 & -6 & x-4 & 2 \\ 4 & -6 & -8 & x+6 \end{pmatrix}$$

$$= \det\begin{pmatrix} x & -x & 4 & -4 \\ -4 & x+2 & 0 & -2 \\ 0 & x-4 & x-4 & 0 \\ 0 & x-4 & -8 & x+4 \end{pmatrix}$$

$$= \det\begin{pmatrix} x & -x & 4+x & -4 \\ -4 & x+2 & -x-2 & -2 \\ 0 & x-4 & 0 & 0 \\ 0 & x-4 & -4-x & x+4 \end{pmatrix}$$

$$= (4-x)\det\begin{pmatrix} x & 4+x & -4 \\ -4 & -x-2 & -2 \\ 0 & -4-x & x+4 \end{pmatrix}$$

$$= (4-x)\det\begin{pmatrix} x & x & -4 \\ -4 & -x-4 & -2 \\ 0 & 0 & x+4 \end{pmatrix}$$

$$= (4-x)\det\begin{pmatrix} x & x & -4 \\ -4 & -x-4 & -2 \\ 0 & 0 & x+4 \end{pmatrix}$$

$$= (4-x)(x+4)\det\begin{pmatrix} x & x \\ -4 & -x-4 \end{pmatrix}$$

$$= x(4-x)(x+4)\det\begin{pmatrix} 1 & 1 \\ -4 & -x-4 \end{pmatrix}$$

$$= x(4-x)(x+4)\det\begin{pmatrix} 1 & 1 \\ 0 & -x \end{pmatrix}$$

$$= -x^2(4-x)(x+4) = x^2(x-4)(x+4)$$

$Sp(A) = \{0, 4, -4\}$.

$ma_A(0) = 2$, $ma_A(4) = ma_A(-4) = 1$

$\Rightarrow mg_A(4) = mg_A(-4) = 1$.

$mg_A(0) = \dim \operatorname{Ker} A = 4 - \operatorname{rg} A$

$$A = \begin{pmatrix} 4 & -2 & -4 & 6 \\ 4 & -2 & 0 & 2 \\ -4 & 6 & 4 & -2 \\ -4 & 6 & 8 & -6 \end{pmatrix}$$

$$\det A = 0$$

$$P_A(x) = (x-\lambda_1)^{m_1} \cdots (x-\lambda_k)^{m_k} = x^n + \underbrace{\cdots}_{x^{n-1}\,\mathrm{Tr}A} + (-1)^n \det A$$

$$(-1)^n \det A = \lambda_1^{m_1} \lambda_2^{m_2} \cdots \lambda_k^{m_k} (-1)^n$$

$\Rightarrow$ $\det A$ = prodotto degli autovalori.

$$\mathrm{Tr}\, A = m_1\lambda_1 + m_2\lambda_2 + \cdots + m_k\lambda_k.$$

$$0 < \underbrace{mg_A(0)}_{4-\mathrm{rg}A} \leq ma_A(0) = 2$$

$$mg_A(0) \in \{1, 2\} \Rightarrow \mathrm{rg}\, A \in \{2, 3\}$$

Il minore 2×2 $\det\begin{pmatrix} -4 & 6 \\ 0 & 2 \end{pmatrix} \neq 0$

$$\Rightarrow \mathrm{rg}\, A \geq 2 \Rightarrow$$

$$\det\begin{pmatrix} -2 & -4 & 6 \\ -2 & 0 & 2 \\ 6 & 4 & -2 \end{pmatrix} = (-2)^3 \det\begin{pmatrix} 1 & 2 & -3 \\ 1 & 0 & -1 \\ -3 & -2 & 1 \end{pmatrix} = -8 \det\begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 0 \\ -3 & -2 & -2 \end{pmatrix}$$

$$= 8 \det\begin{pmatrix} 2 & -2 \\ -2 & -2 \end{pmatrix} = 8 \cdot 4 \det\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} = 32 \det\begin{pmatrix} 1 & -1 \\ 0 & -2 \end{pmatrix} = -64 \neq 0$$

$rg A = 3 \Rightarrow mg_A(0) = 1 \Rightarrow A$ non è diagonalizzabile.

Matrici associate:

$$V \xrightarrow{\mathcal{L}} W \qquad \text{lineare.}$$

$\mathcal{B}_1 \subset V$ base, $\mathcal{B}_2 \subset W$ base.

La matrice che rappresenta $\mathcal{L}$ in $\mathcal{B}_1$ e $\mathcal{B}_2$ è la matrice A che rende commutativo il seguente diagramma

$$\begin{tikzcd} V \arrow[r, "\mathcal{L}"] \arrow[d, "F_{\mathcal{B}_1}"'] & W \arrow[d, "F_{\mathcal{B}_2}"] \\ \mathbb{K}^n \arrow[r, "S_A"] & \mathbb{K}^m \end{tikzcd}$$

Es: $V = \mathbb{R}[x]_{\leq 2}$. $\mathcal{E} = (1, x, x^2)$

$$p(x) = 2 - 3x + 4x^2$$

$$F_{\mathcal{E}}(p) = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

$$p = 2 \cdot 1 - 3 \cdot x + 4 \cdot x^2$$

$$\mathcal{B} = (v_1, v_2, v_3) \overset{\text{base}}{\subset} V \ni v$$

$$v = x_1 v_1 + x_2 v_2 + x_3 v_3$$

$$F_{\mathcal{B}}(v) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{tikzcd} V \arrow[r, "\mathcal{L}"] \arrow[d, shift right, "F_{\mathcal{B}_1}"'] & W \arrow[d, "F_{\mathcal{B}_2}"] \\ \mathbb{K}^n \arrow[u, shift right, "F_{\mathcal{B}_1}^{-1}"'] \arrow[r, dashed, "S_A"] & \mathbb{K}^m \end{tikzcd}$$

$$v_1 = 1 v_1 + 0 v_2 + 0 v_3 + \dots + 0 v_n$$

$$F_{\mathcal{B}_1}(v) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = e_1$$

$$\mathcal{B}_1 = (v_1, \dots, v_n) \qquad \mathcal{B}_2 = (w_1, \dots, w_m)$$

$$A^1 = S_A(e_1) = A e_1 =$$

$$= (F_{\mathcal{B}_2} \circ \mathcal{L} \circ F_{\mathcal{B}_1}^{-1})(e_1)$$

$$= (F_{\mathcal{B}_2} \circ \mathcal{L})(F_{\mathcal{B}_1}^{-1}(e_1)) = (F_{\mathcal{B}_2} \circ \mathcal{L})(v_1)$$

$$= F_{\mathcal{B}_2}(\mathcal{L}(v_1))$$

Es: $\mathcal{L}: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ è l'unica f.ne lineare t.c.

$$\mathcal{L}\left(\begin{pmatrix}1\\1\\1\end{pmatrix}\right) = \begin{pmatrix}1\\1\end{pmatrix}$$

$$\mathcal{L}\left(\begin{pmatrix}1\\1\\0\end{pmatrix}\right) = \begin{pmatrix}1\\-1\end{pmatrix}$$

$$\mathcal{L}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$$

$\left(\left(\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}\right) \text{ è una base di } \mathbb{R}^3\right).$

Scrivere la matrice associata a $\mathcal{L}$ nella base canonica.

__Sol.__: $\mathcal{L}(e_1) = ?$ $\quad \mathcal{L}(e_2) = ?$ $\quad \mathcal{L}(e_3) = ?$

$$\mathcal{B} = \left(\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}\right) \qquad e = (e_1, e_2) \subset \mathbb{R}^2$$

$$\begin{tikzcd}
\mathbb{R}^3 \arrow[r, equal] \arrow[d, "F_e"'] & \mathbb{R}^3 \arrow[r, "\mathcal{L}"] \arrow[d, "F_{\mathcal{B}}"] & \mathbb{R}^2 \arrow[d, equal, "F_e"] \\
\mathbb{R}^3 \arrow[r, shift left, "B^{-1}"] & \mathbb{R}^3 \arrow[l, shift left] \arrow[r, "S_A"] & \mathbb{R}^2
\end{tikzcd}$$

$$A = \begin{pmatrix}1 & 1 & 2\\ 1 & -1 & 1\end{pmatrix}$$

la matrice cercata è AB^{-1}

$$B = \begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}$$

Valutazione:

$$V_{x^2+1}\left(x^3-x+1\right)=$$

$$=\left(x^2+1\right)^3-\left(x^2+1\right)+1$$

$$V_{x^2+1}\left(1\right)=$$

$$V_{x^2+1}\left(1+0x+0x^2\right)=$$

$$=1+0\left(x^2+1\right)+0\left(x^2+1\right)^2=1$$

$$x^2 \longmapsto 2x \xmapsto[\mathrm{Val}_{x^2+1}]{} 2\left(x^2+1\right)$$