$Ax = b$

$(A|b) \rightsquigarrow$ a scala
- b è dominante ①
- b non è dominante ②

① $\Rightarrow$ Non risolubile

②

$\rightsquigarrow \text{rref}(A|b) = (R|d)$

Il sistema $Ax=b$ è equivalente a $Rx=d$.

$$(A|b) \underset{R}{\rightsquigarrow} (R|d)$$

$\exists\, C$ invertibile t.c. $R = CA$ e $d = Cb$

$\bar{x}$ soluzione di $Ax=b$ $\quad (A\bar{x}=b)$

$$R\bar{x} = (CA)\bar{x} = C(A\bar{x}) = Cb = d$$

Viceversa se $R\bar{x} = d$ allora

$$(CA)\bar{x} = Cb \qquad C(A\bar{x}) = Cb \underset{C^{-1}}{\Longrightarrow} A\bar{x} = b$$

$$\begin{cases} \underline{x_1} + 2x_2 + 7x_4 + 5x_6 = 1 \\ \underline{x_3} + 3x_4 + 2x_6 = 3 \\ \underline{x_5} + 2x_6 = 4 \end{cases}$$

$$\boxed{\begin{array}{c} \text{MATLAB} \\ X_0 = A \backslash b \end{array}}$$

$$\begin{cases} x_1 = -2x_2 - 7x_4 - 5x_6 + 1 \\ x_3 = -3x_4 - 2x_6 + 3 \\ x_5 = -2x_6 + 4 \end{cases}$$

$$\text{Soluzioni} = \left\{ \begin{pmatrix} -2x_2 - 7x_4 - 5x_6 + 1 \\ x_2 \\ -3x_4 - 2x_6 + 3 \\ x_4 \\ -2x_6 + 4 \\ x_6 \end{pmatrix} \;\middle|\; x_2, x_4, x_6 \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 4 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -7 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_6 \begin{pmatrix} -5 \\ 0 \\ -2 \\ 0 \\ -2 \\ 1 \end{pmatrix} \;\middle|\; x_2, x_4, x_6 \in \mathbb{R} \right\}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \\ 4 \\ 0 \end{pmatrix} + \left\langle \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -7 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ -2 \\ 0 \\ -2 \\ 1 \end{pmatrix} \right\rangle$$

Es 5.2: (Emiliano - Chimica)

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 1 & 1 & 2 \\ 4 & 1 & 1 & -2 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & -1 & 0 \\ 0 & -7 & -3 & -6 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & -1 & 0 \\ 0 & -1 & -1 & -6 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 0 & 2 & 18 \\ 0 & -1 & -1 & -6 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & 9 \end{array}\right) \rightsquigarrow \left(\begin{array}{ccc|c} 1 & 2 & 0 & -8 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 9 \end{array}\right)$$

$$\rightsquigarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 9 \end{array}\right)$$

Il sistema ammette l'unica soluzione

$$X_0 = \begin{pmatrix} -2 \\ -3 \\ 9 \end{pmatrix}$$

Es 2 : [Emiliano - Chimica]

$\mathcal{B} = (P_1, P_2, P_3, P_4)$: $F(P_1) =$

$$F(P_1) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad F(P_2) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad F(P_3) = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad F(P_4) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Quindi

$$P_1(x) = -\frac{1}{12}(x+1)(x-1)(x-2)$$

$$P_2(x) = -\frac{1}{6}(x+2)(x-1)(x-2)$$

$$P_3(x) = \frac{1}{6}(x+2)(x+1)(x-2)$$

$$P_4(x) = \frac{1}{12}(x+2)(x+1)(x-1)$$

3)

$$\begin{array}{ccc} V & \xrightarrow{L} & V \\ F = F_{\mathcal{B}} \downarrow & & \downarrow F_{\mathcal{B}} = F \\ \mathbb{R}^4 & \xrightarrow{S_A} & \mathbb{R}^4 \end{array} \qquad S_A = F \circ L \circ F^{-1}$$

$$L(P_1) = -\frac{1}{12}\left[(x+2)x(x-1) + x(x-2)(x-3)\right] \Rightarrow F \circ L(P_1) = -\frac{1}{12}\begin{pmatrix} -40 \\ -10 \\ 4 \\ 8 \end{pmatrix}$$

$$L(P_2) = -\frac{1}{6}\left[(x+3)x(x-1) + (x+1)(x-2)(x-3)\right].$$

$$\Rightarrow F \circ L(P_2) = -\frac{1}{6}\begin{pmatrix} -14 \\ 4 \\ 4 \\ 10 \end{pmatrix}$$

$$L(p_3) = \frac{1}{6}\left[(x+3)(x+2)(x-1) + (x+1)x(x-3)\right]$$

$$F \circ L(p_3) = \frac{1}{6}\begin{pmatrix} -10 \\ -4 \\ -4 \\ 14 \end{pmatrix}$$

$$L(p_4) = \frac{1}{12}\left[(x+3)(x+2)x + (x+1)x(x-2)\right]$$

$$F \circ L(p_4) = \frac{1}{12}\begin{pmatrix} -8 \\ -2 \\ 10 \\ 40 \end{pmatrix}$$

$$A = \begin{pmatrix} 10/3 & 7/3 & -5/3 & -2/3 \\ 5/6 & -2/3 & -2/3 & -1/6 \\ -1/3 & -2/3 & -2/3 & 5/6 \\ -2/3 & -5/3 & 7/3 & 10/3 \end{pmatrix}$$