Es 1:

$$D = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 2 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 3 \end{pmatrix} \qquad D' = \begin{pmatrix} -1 & \\ & -2 \end{pmatrix}$$

$$DA = \begin{pmatrix} 1 & 1 \\ -2 & -2 \\ 6 & 6 \end{pmatrix} \qquad AD' = \begin{pmatrix} -1 & -2 \\ -2 & -4 \\ -3 & -6 \end{pmatrix}$$

Moltiplicare a sinistra per una matrice diagonale ha l'effetto di moltiplicare le righe per gli elementi della diagonale

Moltiplicare a destra per una matrice diagonale ha l'effetto di moltiplicare le colonne per gli elementi della diagonale.

Es2: Ricordiamo che $A, B \in Mat_{m \times n}(\mathbb{K})$ sono *simili* se e solo se $rg(A) = rg(B)$.

In questo caso esistono basi $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4$ t.c.

$$
\begin{tikzcd}
\mathbb{K}^n \arrow[r, "S_A"] \arrow[d, "F_{\mathcal{B}_1}"'] & \mathbb{K}^m \arrow[d, "F_{\mathcal{B}_2}"] \\
\mathbb{K}^n \arrow[r, "\begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix}"'] & \mathbb{K}^m
\end{tikzcd}
\qquad
\begin{tikzcd}
\mathbb{K}^n \arrow[r, "S_B"] \arrow[d, "F_{\mathcal{B}_3}"'] & \mathbb{K}^m \arrow[d, "F_{\mathcal{B}_4}"] \\
\mathbb{K}^n \arrow[r, "\begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix}"'] & \mathbb{K}^m
\end{tikzcd}
$$

$\Rightarrow$

$$
F_{\mathcal{B}_2}^{-1} \circ F_{\mathcal{B}_1} = F_1
\quad
\begin{tikzcd}
\mathbb{K}^n \arrow[r, "A"] \arrow[d, "F_{\mathcal{B}_1}"'] \arrow[dd, bend right=60] & \mathbb{K}^m \arrow[d, "F_{\mathcal{B}_2}"] \arrow[dd, bend left=60] \\
\mathbb{K}^n \arrow[r, "\begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & 0 \end{pmatrix}"] & \mathbb{K}^m \\
\mathbb{K}^n \arrow[r, "B"] \arrow[u, "F_{\mathcal{B}_3}"] & \mathbb{K}^m \arrow[u, "F_{\mathcal{B}_4}"']
\end{tikzcd}
\quad
F_2 = F_{\mathcal{B}_4}^{-1} \circ F_{\mathcal{B}_2}
$$

1) $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ $B = \mathbb{1}_2$.

$\operatorname{rg} A = 2 = \operatorname{rg} B$

$$\begin{tikzcd} \mathbb{C}^2 \arrow[r, "A"] \arrow[d, "\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}"'] & \mathbb{C}^2 \arrow[d, "A^{-1} = F_{\mathcal{B}}"] \\ \mathbb{C}^2 \arrow[r, "\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}"'] & \mathbb{C}^2 \end{tikzcd}$$

$\mathcal{B} = \{ A^1, A^2 \}$

$A: \; e_1 \longmapsto \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad A^{-1}: \begin{pmatrix} 1 \\ 1 \end{pmatrix} \longmapsto e_1$

$ \; e_2 \longmapsto \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\quad \phantom{A^{-1}:} \begin{pmatrix} 1 \\ 2 \end{pmatrix} \longmapsto e_2$

$S_{A^{-1}}(e_1) = ?$ $\quad e_1 = 2\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow A^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

$e_2 = -\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

2) $\operatorname{rg} A = 2$, $\operatorname{rg} B = 1$ $\Rightarrow$ A e B non sono simili.

3) $A = \begin{pmatrix} 1 & 1+i \\ 2 & 2+2i \end{pmatrix}$ $\qquad B = \begin{pmatrix} 1+i & 1-i \\ i & 1 \end{pmatrix}$

$\operatorname{rg} A = 1 = \operatorname{rg} B$

$A^2 = (1+i)\, A^1 \Rightarrow$

$(1+i)A^1 - A^2 = 0$

$A \begin{pmatrix} 1+i \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$B^1 = i\, B^2$

$$
\begin{tikzcd}
\mathbb{C}^2 \arrow[r, "A"] \arrow[d, "\begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix}^{-1}"'] & \mathbb{C}^2 \arrow[d, "\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1}"] \\
\mathbb{C}^2 \arrow[r, "\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}"] & \mathbb{C}^2 \\
\mathbb{C}^2 \arrow[r, "\begin{pmatrix} 1+i & 1-i \\ i & 1 \end{pmatrix}"] \arrow[u, "\begin{pmatrix} 1 & 1 \\ 0 & -i \end{pmatrix}^{-1}"] & \mathbb{C}^2 \arrow[u, "\begin{pmatrix} 1+i & 0 \\ i & 1 \end{pmatrix}^{-1}"']
\end{tikzcd}
$$

$F_1 = \begin{pmatrix} 1 & 1 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix}^{-1} = ?$ $\qquad F_2 = \begin{pmatrix} 1+i & 0 \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1}$

$$F_1 = \begin{pmatrix} 1 & 1 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix}^{-1} = ? \qquad F_2 = \begin{pmatrix} 1+i & 0 \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1}$$

$$\begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix} \qquad e_2 = +(1+i)\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1+i \\ -1 \end{pmatrix}$$

$$F_1 = \begin{pmatrix} 1 & 1 \\ 0 & -i \end{pmatrix} \begin{pmatrix} 1 & 1+i \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & i \\ 0 & i \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \qquad e_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$F_2 = \begin{pmatrix} 1+i & 0 \\ i & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 1+i & 0 \\ i-2 & 1 \end{pmatrix}$$

Da verificare: $\quad F_2 A = B F_1$

4)

$$A = \begin{pmatrix} 1 & 1+i & 1 \\ i & -1+i & 1 \\ -i & 1-i & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$A^2 = (1+i)\, A^1 = (1+i) \begin{pmatrix} 1 \\ i \\ -i \end{pmatrix} = \begin{pmatrix} 1+i \\ i-1 \\ -i+1 \end{pmatrix}$$

$\{A^1, A^3\}$ sono lin. ind. $\quad$ $\mathrm{rg}\, A = 2 = \mathrm{rg}\, B$.

$$F_1 = \begin{array}{c} \begin{pmatrix} 1 & 0 & 1+i \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}^{-1} \\ \begin{matrix} v_1 & v_2 & v_3 \end{matrix} \end{array} \qquad \begin{tikzcd} \mathbb{C}^3 \arrow[r, "A"] \arrow[d] & \mathbb{C}^3 \arrow[d] \\ \mathbb{C}^3 \arrow[r, "B"] & \mathbb{C}^3 \end{tikzcd} \qquad \begin{array}{c} \begin{pmatrix} 1 & 1 & 0 \\ i & 1 & 0 \\ -i & 1 & 1 \end{pmatrix}^{-1} \\ \begin{matrix} w_1 & w_2 & w_3 \end{matrix} \end{array} = F_2$$

$e_1 = x_1($

$e_1 = v_1$

$$e_2 = (1+i) \overset{v_1}{e_1} - \underbrace{\begin{pmatrix} 1+i \\ -1 \\ 0 \end{pmatrix}}_{v_3}$$

$e_3 = v_2$

$$F_1 = \begin{pmatrix} 1 & 1+i & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\begin{array}{c} \begin{pmatrix} 1 & 1 & 0 \\ i & 1 & 0 \\ -i & 1 & 1 \end{pmatrix}^{-1} \\ \begin{matrix} w_1 & w_2 & w_3 \end{matrix} \end{array} = F_2 \quad = \begin{pmatrix} \frac{1+i}{2} \\ \frac{1-i}{2} \\ i-1 \end{pmatrix}$$

$$e_1 = x_1 w_1 + x_2 w_2 + x_3 w_3 = \begin{pmatrix} x_1 + x_2 \\ i x_1 + x_2 \\ -i x_1 + x_2 + x_3 \end{pmatrix}$$

$$x_1 + x_2 = 1 \qquad x_1 - i x_1 = 1 \Rightarrow x_1 (1-i) = 1$$

$$i x_1 + x_2 = 0 \Rightarrow x_2 = -i x_1$$

$$-i x_1 + x_2 + x_3 = 0 \qquad -i x_1 - i x_1 + x_3 = 0 \qquad x_3 = 2i x_1$$

$$x_1 = \frac{1+i}{2} \qquad (a+ib)^{-1} = \frac{a-ib}{a^2+b^2}$$

$$x_1 = \frac{1}{2} + \frac{i}{2}, \quad x_2 = -i \frac{1+i}{2} = \frac{-i+1}{2} = \frac{1}{2} - \frac{1}{2} i$$

$$x_3 = i(1+i) = i-1$$

$$\begin{array}{c}\begin{pmatrix} 1 & 1 & 0 \\ i & 1 & 0 \\ -i & 1 & 1 \end{pmatrix}^{-1} \\ \begin{array}{ccc} w_1 & w_2 & w_3 \end{array}\end{array} = F_2 = \begin{pmatrix} \frac{1+i}{2} & & 0 \\ \frac{1-i}{2} & & 0 \\ i-1 & & 1 \end{pmatrix}$$

$$e_2 = x_1 w_1 + x_2 w_2 + x_3 w_3 = \dots \qquad \text{Esercizio}$$

Es 3 :

1) F : ~~A=~~(Es 1 DA ~~ed~~ è def. ma AD no.

2) V

3) F : $A=\begin{pmatrix}1&2\\3&4\end{pmatrix}$ $B=\begin{pmatrix}1&6\\5&7\end{pmatrix}$ $AB=\begin{pmatrix}11\end{pmatrix.$ $\neq$ $BA=\begin{pmatrix}18\end{pmatrix.$

4) V

5) F : $A=\begin{pmatrix}0&1\\0&0\end{pmatrix}$ $B=\begin{pmatrix}2&3\\4&5\end{pmatrix}$ $AB=\begin{pmatrix}4&5\\0&0\end{pmatrix}$

6) F : $\underset{A}{(1,-1)}\underset{B}{\begin{pmatrix}1\\1\end{pmatrix}}=0$

7) F : vedi 3

8) F : Basta prendere $A=0$. (Vero se A è invertibile)

9) F : $\underset{A}{(1,0)}\underset{B}{\begin{pmatrix}1\\0\end{pmatrix}}=1$ (Vero se A è quadrata)

Es 4: 1) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Es 5:

$$T^n = \begin{cases} T & \text{se } n \text{ è dispari} \\ T^2 & \text{se } n \text{ è pari} \end{cases}$$