Esercizi settimanali:

1) $\{X\in\mathbb{R}^3 \mid x_1+x_2-x_3=1\}$ ✗

$\{X\in\mathbb{R}^3 \mid x_1+x_2-x_3=0\} = \left\langle \begin{pmatrix}-1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix} \right\rangle$ ✓

$\{X\in\mathbb{R}^3 \mid x_1^2+x_2-x_3=0\} \ni \quad \begin{pmatrix}1\\1\\2\end{pmatrix}+\begin{pmatrix}2\\2\\6\end{pmatrix} = \begin{pmatrix}3\\3\\8\end{pmatrix} \notin \Rightarrow$ Non è.

$\{X\in\mathbb{R}^3 \mid x_1+x_2-x_3=0,\ x_1\geq 0\} \ni \begin{pmatrix}1\\0\\1\end{pmatrix}$ ma $\begin{pmatrix}-1\\0\\-1\end{pmatrix} \notin \Rightarrow$ Non è

$\{X\in\mathbb{R}^3 \mid x_1\geq 0,\ x_2+x_3=0\} \ni \begin{pmatrix}1\\0\\0\end{pmatrix}$ ma $\begin{pmatrix}-1\\0\\0\end{pmatrix} \notin \Rightarrow$ Non è

$\{X\in\mathbb{R}^3 \mid x_1x_2x_3=0\} \ni \begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}$ ma $\begin{pmatrix}1\\1\\0\end{pmatrix}+\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}1\\2\\1\end{pmatrix} \notin \Rightarrow$ Non è.

$\{X\in\mathbb{R}^3 \mid x_1+x_2-x_3=0,\ x_2+x_3=0\} = \left\langle \begin{pmatrix}2\\-1\\1\end{pmatrix} \right\rangle$

$\{X\in\mathbb{R}^3 \mid x_1+x_2-x_3=0,\ x_2+x_3=1\}$ ✗

ES 2: Calcolare $U \cap W$: $(2 \neq 0)$

1) $U = \langle u, u+v \rangle$, $W = \langle 2u+v \rangle$
2) $U = \langle u+v, u-v \rangle$, $W = \langle u, v \rangle$
3) $U = \langle u+v \rangle$, $W = \langle u+2v \rangle$

Sol.: 1) $2u+v = u + (u+v) \in U \Rightarrow W \subseteq U \Rightarrow U \cap W = W$

2) $U \subseteq W \Rightarrow U \cap W \supseteq U \Rightarrow U \cap W = U$.

$$\left.\begin{array}{l} 2u = (u+v) + (u-v) \Rightarrow u \in U \\ 2v = (u+v) - (u-v) \Rightarrow v \in U \end{array}\right\} \Rightarrow W \subseteq U \subseteq W \Rightarrow U = W.$$

$\langle u, v \rangle = \langle u+v, v \rangle = \langle u+v, u-v \rangle$ (lemma di scambio)

3) Se $v = 0_V$, $U = W$. Se $u = 0_V$, $U = W$.

Se $u \neq 0_V$ e $v \neq 0_V$? $u+v = t(u+2v) \Leftrightarrow (1-t)u = (2t-1)v$

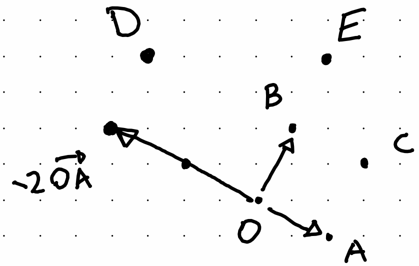
Se $t = 1$ allora $v = 2v$ e quindi $(2 \neq 0)$ $v = 0_V$ ↯

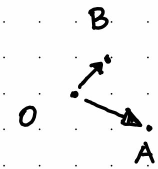
Se $t \neq 1$ $u = \dfrac{(2t-1)}{1-t} v \in \langle v \rangle$ $U \cap W = \begin{cases} U = W & \text{se lin. Dip.} \\ \{0_V\} & \text{altrimenti.} \end{cases}$

Es3 : 1) Calcolare $\overrightarrow{OA}+\overrightarrow{OB}$, $(-2)\overrightarrow{OA}+\overrightarrow{OB}$, $2\overrightarrow{OB}$

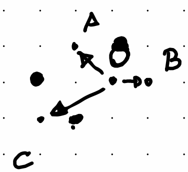
$$\overrightarrow{OA}+\overrightarrow{OB} = \overrightarrow{OC}, \quad (-2)\overrightarrow{OA}+\overrightarrow{OB} = \overrightarrow{OD}, \quad 2\overrightarrow{OB} = \overrightarrow{OE}$$

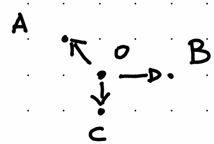
1)





2) $(\overrightarrow{OA}+\overrightarrow{OB})+\overrightarrow{OC} = \overrightarrow{OA}+(\overrightarrow{OB}+\overrightarrow{OC})$, $2(\overrightarrow{OA}+\overrightarrow{OB}) = 2\overrightarrow{OA}+2\overrightarrow{OB}$





Es 4: $\{v_1, v_2, v_3\}$ è lin. Ind.?

1) $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ LIN. DIP. (Per il Teo fond.).

2) $v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ Lin. Ind.

3) $v_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ Lin Ind.

4) $v_1 = 1+x$, $v_2 = 1+x-x^2$, $v_3 = 1+x+x^3$ Lin Ind.

5) $v_1 = \sin(x)$, $v_2 = \sin(2x)$, $v_3 = \sin(3x)$. Lin Ind.

Sol.: $t_1 \sin(x) + t_2 \sin(2x) + t_3 \sin(3x) \equiv 0 \quad \forall x \in \mathbb{R}$

$x = \pi/2 \qquad t_1 + 0 - t_3 = 0$

$x = \pi/3 \qquad t_1 \frac{\sqrt{3}}{2} + t_2 \frac{\sqrt{3}}{2} + \ = 0$

$$\begin{cases} t_1 = t_3 \\ t_2 = -t_1 = -t_3 \end{cases}$$

$t_1 = t_2 = 0 \Longleftarrow$

$t_3 = 0$

$\Uparrow$

$x = \pi/6 \qquad t_1 \frac{1}{2} + t_2 \frac{\sqrt{3}}{2} + t_3 = 0 \longrightarrow t_1 + \sqrt{3}\, t_2 + t_3 = 0$

$\Rightarrow t_3 - \sqrt{3}\, t_3 + t_3 = 0 \Rightarrow (2-\sqrt{3})\, t_3 = 0$

Es 5:

$\mathcal{B}_1 = \left\{ \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$, $\mathcal{B}_2 = \left\{ \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\}$, $\mathcal{B}_3 = \left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix} \right\}$

$v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $v_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, $v_3 = \begin{pmatrix} 7 \\ -7 \end{pmatrix}$, $v_4 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Sol.:

Verifica: $-2\begin{pmatrix} 2 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ ✓

$$F_{\mathcal{B}_1}(v_1) = \begin{pmatrix} -2 \\ 5 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_1) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$F_{\mathcal{B}_1}(v_2) = \begin{pmatrix} 3 \\ -4 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_2) = \begin{pmatrix} -8/3 \\ 7/3 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_2) = \begin{pmatrix} -3/2 \\ -1/2 \end{pmatrix}$$

$$F_{\mathcal{B}_1}(v_3) = \begin{pmatrix} 14 \\ -21 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_3) = \begin{pmatrix} -35/3 \\ 28/3 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_3) = \begin{pmatrix} -7 \\ 0 \end{pmatrix}$$

$$F_{\mathcal{B}_1}(v_4) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \quad F_{\mathcal{B}_2}(v_4) = \begin{pmatrix} -1/3 \\ 2/3 \end{pmatrix}; \quad F_{\mathcal{B}_3}(v_4) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$