$$A = \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix} \qquad \ker b = \left\langle \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\rangle = u_3$$

$$e_1^2 = a_{11} = -1 < 0$$

$$\langle e_1 \rangle^\perp = \{ x \mid -x_1 + x_2 - x_3 = 0 \} = \left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\rangle$$

$$u_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \qquad u_2^2 = (1,1,0) \begin{pmatrix} -1 & 1 & -1 \\ 1 & 1 & 1 \\ -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} =$$

$$= (0,2,0) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 2 > 0$$

Quindi

$\mathcal{B}' : \{ u_2, e_1, u_3 \}$

è una base ortogonale di $(\mathbb{R}^3, b_A)$

$$sg(b) = (1,1)$$

$$\mathcal{S} = \left\{ E_1 = \frac{u_2}{\sqrt{u_2^2}} = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}, E_2 = \frac{e_1}{\sqrt{-e_1^2}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, E_3 = u_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right\}$$

è una base di Sylvester.

$$S = \begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix}$$

v è isotropo se $b(v,v)=0$

$$b(X,Y) = (x_1+x_2+x_3)(y_1+y_2+y_3) - 2(x_1+x_3)(y_1+y_3)$$

$$v = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$b(v,v) = (x_1+x_2+x_3)^2 - 2(x_1+x_3)^2$$

$$= (x_1+x_2+x_3)^2 - \sqrt{2}^2 (x_1+x_3)^2$$

$$= \left(x_1+x_2+x_3 + \sqrt{2}(x_1+x_3)\right)\left(x_1+x_2+x_3 - \sqrt{2}(x_1+x_3)\right)$$

$a^2-b^2 = (a+b)(a-b)$

$b(v,v)=0 \iff (1+\sqrt{2})x_1 + x_2 + (1+\sqrt{2})x_3 = 0 : \pi_1$

oppure

$(1-\sqrt{2})x_1 + x_2 + (1-\sqrt{2})x_3 = 0 : \pi_2$

vettori isotropi $= \pi_1 \cup \pi_2$.

(Non sono un sottospazio vettoriale!)

Es2 : Le risposte sono:

$sg(A) = (2,0)$

$sg(B) = (2,1)$

$sg(C) = (1,1)$

$sg(D) = (2,1)$

B e D sono congruenti.

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\ker A = \langle \underbrace{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}}_{=v} \rangle$, $U = \langle e_1, e_3 \rangle$

$e_1^2 = a_{11} = 1 > 0$. $b_A(e_1, e_3) = a_{13} = 0$

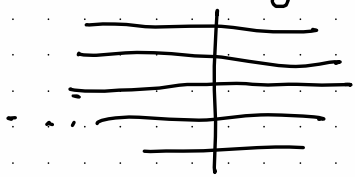
$e_3^2 = a_{33} = 1 > 0$

$\{e_1, e_3, v\}$ è una base ortogonale di $(\mathbb{R}^3, b_A)$.

$sg(A) = sg(b) = (2,0)$

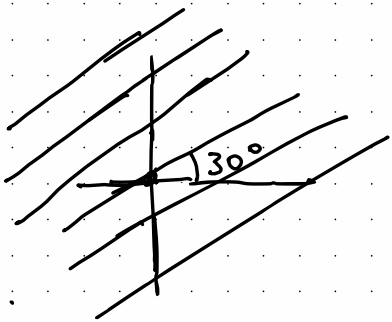
Es3:

1) $(1,0)$: $y=0$, $r = \left\langle \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\rangle$



$y=c$

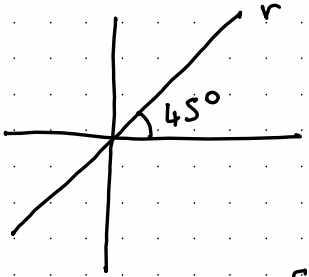
2) $r = \left\langle \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} \right\rangle$ (a = 1/2, b = $\sqrt{3}/2$)



$r: \frac{1}{2}x - \frac{\sqrt{3}}{2}y = 0$

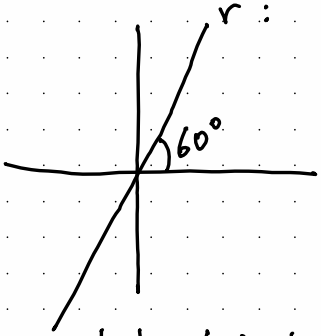
$\frac{1}{2}x - \frac{\sqrt{3}}{2}y = c$

3)



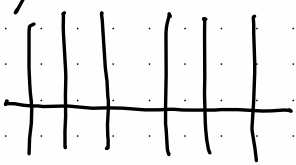
$r: \quad x - y = c$

4)



$r: \quad \frac{\sqrt{3}}{2}x - \frac{1}{2}y = c$

5)



$x = c$

Es4:

$r_1: \quad 3x - 5y + 2 = 0$

$\pm \frac{1}{\sqrt{34}} \begin{pmatrix} 5 \\ 3 \end{pmatrix}$: versori direttori

$\pm \frac{1}{\sqrt{34}} \begin{pmatrix} 3 \\ -5 \end{pmatrix}$: versori normali

Es5:

$$F_1 = v_1$$

$$F_2 = v_2 - \frac{v_2 \cdot F_1}{F_1 \cdot F_1} F_1$$

$$F_3 = v_3 - \frac{v_3 \cdot F_1}{F_1 \cdot F_1} F_1 - \frac{v_3 \cdot F_2}{F_2 \cdot F_2} F_2$$

$$F_4 = v_4 - \frac{v_4 \cdot F_1}{F_1 \cdot F_1} F_1 - \frac{v_4 \cdot F_2}{F_2 \cdot F_2} F_2 - \frac{v_4 \cdot F_3}{F_3 \cdot F_3} F_3$$

$$E_i = \frac{F_i}{\| F_i \|}$$

$$\mathcal{B}' = \left\{ \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1/\sqrt{6} \\ 1/\sqrt{6} \\ 0 \\ \sqrt{2}/\sqrt{3} \end{pmatrix}, \begin{pmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 0 \\ 1/\sqrt{3} \end{pmatrix} \right\}$$