Es 6 : r: retta, π: piano di $\mathbb{R}^3$.

1) $r = X_0 + \langle v \rangle$, π : $AX = b$

$$Av = 0 \overset{def}{\iff} r \text{ e } \pi \text{ sono parallele}$$

$$\exists\, t : A(X_0 + tv) = b \iff r \cap \pi \neq \emptyset .$$

$t\,Av = b - AX_0$. ha soluzione.

	$rg(Av)$	$rg(Av \mid b - AX_0)$
$r \subset \pi$	0	0
$r \,//\, \pi$, $r \cap \pi = \emptyset$	0	1
$r \cap \pi = \{P_0\}$ $t_0 := (Av)^{-1}(b - AX_0)$ $P_0 = X_0 + t_0 v$	1	1

2) $r = X_0 + \langle v \rangle$, $\pi = Y_0 + \langle w_1, w_2 \rangle$

$r \parallel \pi \iff v \in \langle w_1, w_2 \rangle \iff rg(w_1\, w_2\, v) = 2 \iff \det(w_1\, w_2\, v) = 0$

$r \cap \pi \neq \emptyset \iff \exists\, t \in \mathbb{R}$ t.c. $X_0 + tv \in \pi = Y_0 + \langle w_1, w_2 \rangle$

$\iff \exists\, t, s_1, s_2 \in \mathbb{R}$ t.c. $X_0 + tv = Y_0 + s_1 w_1 + s_2 w_2$

$\iff s_1 w_1 + s_2 w_2 - tv = Y_0 - X_0$

$\iff (w_1\ w_2\ -v)\begin{pmatrix} s_1 \\ s_2 \\ t \end{pmatrix} = Y_0 - X_0$ è risolubile.

	$\det(w_1\, w_2\, v)$	$rg(w_1\, w_2\, v \mid Y_0 - X_0)$
$r \subset \pi$	0	2
$r \parallel \pi$, $r \cap \pi = \emptyset$	0	3
$r \cap \pi = \{P_0\}$	$\neq 0$	3

Sia $\begin{pmatrix} s_1 \\ s_2 \\ t \end{pmatrix} = (w_1, w_2, v)^{-1}(Y_0 - X_0)$

$\Rightarrow P_0 = X_0 - tv$

3) $r: AX = b$, $\pi = Y_0 + \langle w_1, w_2 \rangle$ $\quad rg\, A = 2$, $A \in Mat_{2\times 3}(\mathbb{R})$.

$r \parallel \pi \iff \operatorname{Ker} A \subset \langle w_1, w_2 \rangle \iff \exists \underset{\neq (0,0)}{(t_1, t_2)}$ t.c. $A(t_1 w_1 + t_2 w_2) = 0_2$

$\iff rg\,(Aw_1 | Aw_2) = 1 \underset{\dim \operatorname{Ker} A = 1}{\iff} \det(Aw_1 | Aw_2) = 0$

$r \cap \pi \neq \emptyset \iff \exists\, s_1, s_2$ t.c. $A(Y_0 + s_1 w_1 + s_2 w_2) = b$

$\iff \exists\, s_1, s_2$ t.c. $s_1 Aw_1 + s_2 Aw_2 = b - AY_0$

$\iff (Aw_1 | Aw_2)\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = b - AY_0$ ha soluzione.

	$\det(Aw_1 \mid Aw_2)$	$rg\,(Aw_1 \mid Aw_2 \mid b - AY_0)$
$r \subset \pi$	0	1
$r \parallel \pi$, $r \cap \pi = \emptyset$	0	2
$r \cap \pi = \{P_0\}$:	$\neq 0$	2

$P_0 = Y_0 + s_1 w_1 + s_2 w_2$ con

$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = (Aw_1 | Aw_2)^{-1}(b - AY_0)$

4) $r: A_r X = b_r$ $\quad \pi: A_\pi X = b_\pi$ $\quad \operatorname{rg} A_r = 2$, $A_r \in Mat_{2\times 3}(\mathbb{R})$

$\operatorname{rg} A_\pi = 1$, $A_\pi \in Mat_{1\times 3}(\mathbb{R})$

$r \parallel \pi \iff \operatorname{Ker} A_r \subset \operatorname{Ker} A_\pi$

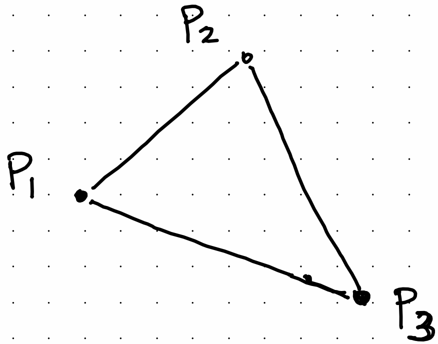
$\iff \exists X \neq 0$ t.c. $\begin{matrix} A_r X = 0 \\ A_\pi X = 0 \end{matrix} \iff \operatorname{Ker} \overbrace{\begin{pmatrix} A_r \\ \hline A_\pi \end{pmatrix}}^{3\times 3} \neq \{0_3\}$.

$\iff \operatorname{rg}\begin{pmatrix} A_r \\ \hline A_\pi \end{pmatrix} = 2 \iff \det\begin{pmatrix} A_r \\ \hline A_\pi \end{pmatrix} = 0$

$r \cap \pi \neq \emptyset \iff \exists X$ t.c. $\begin{cases} A_r X = b_r \\ A_\pi X = b_\pi \end{cases} \iff \operatorname{rg}\begin{pmatrix} A_r \\ \hline A_\pi \end{pmatrix} = \operatorname{rg}\left(\begin{array}{c|c} A_r & b_r \\ \hline A_\pi & b_\pi \end{array}\right)$

	$\det\begin{pmatrix} A_r \\ \hline A_\pi \end{pmatrix}$	$\operatorname{rg}\left(\begin{array}{c\|c} A_r & b_r \\ \hline A_\pi & b_\pi \end{array}\right)$
$r \subset \pi$	0	2
$r \parallel \pi$, $r \cap \pi = \emptyset$	0	3
$r \cap \pi = \{P_0\}$ con $P_0 = \begin{pmatrix} A_r \\ \hline A_\pi \end{pmatrix}^{-1} \begin{pmatrix} b_r \\ \hline b_\pi \end{pmatrix}$	$\neq 0$	3

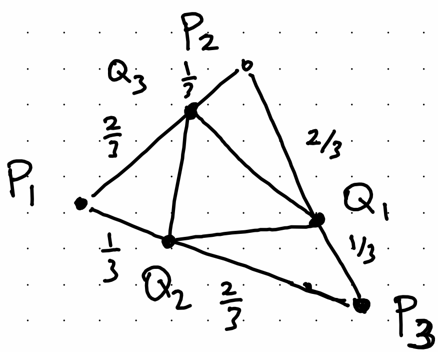
Es 4 :



Ip. : $\frac{1}{2} \left| \det \left(P_2 - P_1 \mid P_3 - P_1 \right) \right| = 1$

1) $P_1 + \langle P_2 - P_1 \rangle$, $P_2 + \langle P_3 - P_2 \rangle$, $P_3 + \langle P_3 - P_1 \rangle$

2)



$$Q_1 = \frac{1}{3} P_3 + \frac{2}{3} P_2$$

$$Q_2 = \frac{2}{3} P_3 + \frac{1}{3} P_1$$

$$Q_3 = \frac{2}{3} P_1 + \frac{1}{3} P_2$$

3) $\text{Area}\left(\overset{\triangle}{Q_1, Q_2 Q_3}\right) = \frac{1}{2} \left| \det \left(Q_1 - Q_2 \mid Q_3 - Q_2 \right) \right|$

$$= \frac{1}{2} \left| \det \left(\frac{1}{3}(P_3 - P_1) + \frac{2}{3}(P_2 - P_3) \;\middle|\; \frac{1}{3}(P_2 - P_1) - \frac{2}{3}(P_3 - P_1) \right) \right|$$

$$= \frac{1}{2} \frac{1}{9} \left| \det \left((P_3 - P_1) + 2(P_2 - P_3) \;\middle|\; (P_2 - P_1) - 2(P_3 - P_1) \right) \right|$$

$$= \frac{1}{2}\,\frac{1}{9}\,\left|\det\left(\left(P_3-P_1\right)+2\left(P_2-P_3\right) \,\middle|\, \left(P_2-P_1\right)-2\left(P_3-P_1\right)\right)\right|$$

$$P_2-P_3 = \left(P_2-P_1\right)+\left(P_1-P_3\right) = \left(P_2-P_1\right)-\left(P_3-P_1\right)$$

$$= \frac{1}{2}\,\frac{1}{9}\,\left|\det\left(-\left(P_3-P_1\right)+2\left(P_2-P_1\right) \,\middle|\, \left(P_2-P_1\right)-2\left(P_3-P_1\right)\right)\right|$$

$$= \frac{1}{2}\,\frac{1}{9}\,\left|\det\left[\left(P_2-P_1 \,\middle|\, P_3-P_1\right)\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}\right]\right|$$

$$= \frac{1}{2}\,\frac{1}{9}\,\left|\det\left(P_2-P_1 \,\middle|\, P_3-1\right)\,\det\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}\right| = \frac{1}{3}$$

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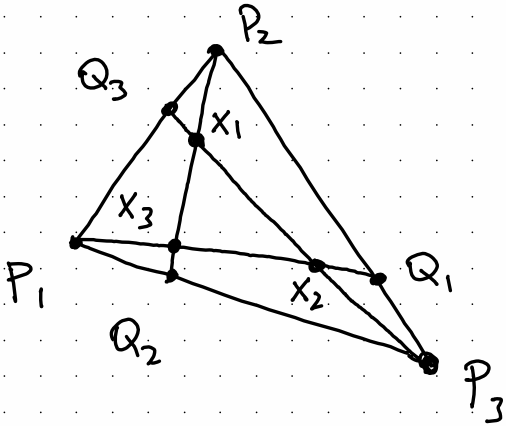
4) $r_1 = P_1 + \langle Q_1 - P_1 \rangle$, $r_2 = P_2 + \langle Q_2 - P_2 \rangle$, $r_3 = P_3 + \langle Q_3 - P_3 \rangle$

5)



$$X_1 = \frac{2}{7} P_1 + \frac{1}{7} P_2 + \frac{4}{7} P_3$$

$$X_2 = \frac{2}{7} P_2 + \frac{1}{7} P_3 + \frac{4}{7} P_1$$

$$X_3 = \frac{2}{7} P_3 + \frac{1}{7} P_1 + \frac{4}{7} P_2$$

$$X_1 = t\, P_2 + (1-t) Q_2 \qquad (t \geq 0)$$

$$= s\, P_3 + (1-s) Q_3 \qquad (s \geq 0)$$

$$t\, P_2 + \frac{2(1-t)}{3} P_3 + \frac{(1-t)}{3} P_1 = s P_3 + \frac{2(1-s)}{3} P_1 + \frac{(1-s)}{3} P_2$$

$$s = \frac{2}{3} - \frac{2}{3} t$$

$$t = \frac{1}{3} - \frac{1}{3} s \qquad \dots$$

6) $\text{Area}\left(\overset{\triangle}{x_1 x_2 x_3}\right) = \frac{1}{2}\left|\det\left(x_2 - x_1 \mid x_3 - x_1\right)\right|$

$= \cdots = \frac{1}{2}\frac{1}{49}\left|\det\left(P_2 - P_1 \mid P_3 - P_1\right)\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}\right| = \frac{1}{7}$