$\boxed{\text{COMPITO C}}$ 17/11/2025

C_1

1) OMOGENEA ASSOCIATA:

$$4\alpha^2+2\alpha=0 \quad \Rightarrow \alpha_1=-\frac{1}{2};\ \alpha_2=0$$

$$\Rightarrow y_0(x)=C_1e^{-\frac{1}{2}x}+C_2$$

NON OMOGENEA: Metodo di somiglianza.
Poichè $\alpha=0$ è radice del polinomio caratteristico, allora

$$y_p(x)=x(Ax+B)=Ax^2+Bx$$

$$y_p'(x)=2Ax+B$$

$$y_p''(x)=2A$$

$$\Rightarrow \quad 8A+4Ax+2B=-2x+1$$

$$\begin{cases}4A=-2\\8A+2B=1\end{cases} \Rightarrow \begin{cases}A=-\frac{1}{2}\\B=\frac{1}{2}-4A=\frac{5}{2}\end{cases}$$

$$\Rightarrow y_{NO}(x)=C_1e^{-\frac{1}{2}x}+C_2-\frac{1}{2}x^2+\frac{5}{2}x$$

$$y(0)=C_1+C_2=0$$

$$y_{NO}'(x)=-\frac{1}{2}C_1e^{-\frac{1}{2}x}-x+\frac{5}{2}$$

$$y'(0)=-\frac{1}{2}C_1+\frac{5}{2}=0$$

$$\Rightarrow \begin{cases}C_1=5\\C_2=-5\end{cases} \Rightarrow y(x)=5e^{-\frac{1}{2}x}-5-\frac{1}{2}x^2+\frac{5}{2}x$$

C_2

2)
$$\exp\left(\frac{1}{x^3}\right)-1-\frac{1}{x^3}=1+\frac{1}{x^3}+\frac{1}{2x^6}+o\left(\frac{1}{x^6}\right)-1-\frac{1}{x^3}$$
$$=\frac{1}{2x^6}+o\left(\frac{1}{x^6}\right) \quad \text{per } x\to+\infty$$

$$\Rightarrow f(x) \underset{x\to+\infty}{\sim} x^\alpha \mathrm{Sh}\left(\frac{1}{2x^6}\right) \underset{x\to+\infty}{\sim} \frac{1}{2x^{6-\alpha}}$$

Pertanto f è integrabile in $[1,+\infty)$ se $6-\alpha>1$ cioè se $\alpha<5$; f non è integrabile in $[1,+\infty)$ se $\alpha\geq 5$

3)
$$\frac{3(n^3+2)}{n^\alpha}\leq a_n\leq\frac{5(n^3+2)}{n^\alpha}$$

Poiché $\frac{(n^3+2)}{n^\alpha} \underset{n\to+\infty}{\sim} \frac{1}{n^{\alpha-3}}$

e $\sum\frac{1}{n^{\alpha-3}}=\begin{cases}\text{converge} & \text{se } \alpha>4\\ \text{diverge} & \text{se } \alpha\leq 4\end{cases}$

allora $\underline{\forall\alpha>4}$ $\quad \sum a_n\leq\sum\frac{5(n^3+2)}{n^\alpha}\sim 5\underbrace{\sum\frac{1}{n^{\alpha-3}}}_{\text{convergente}}$

$\underline{\forall\alpha\leq 4}$ $\quad \sum a_n\geq\sum\frac{3(n^3+2)}{n^\alpha}\sim 3\underbrace{\sum\frac{1}{n^{\alpha-3}}}_{\text{divergente}}$.

4) DOMINIO: $x^2-x+1>0$ vero $\forall x\in\mathbb{R}$

poiché $\Delta<0$.

(S3)

$\Rightarrow D=\mathbb{R}$

$\cap$ asse y: $f(0)=\ln 1=0$

$\cap$ asse x: $f(x)=0 \iff x^2-x+1=1$

$\iff x_1=0$; $x_2=1$.

Segno: $f(x)>0 \iff x^2-x>0 \iff x<0 \vee x>1$.

$\lim_{x\to\pm\infty} f(x)=+\infty$. $\nexists$ asintoti obliqui, in quanto

$$\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{2\ln x}{x}=0.$$

MONOTONIA:

$$f'(x)=\frac{2x-1}{\underbrace{x^2-x+1}_{>0}}\geq 0 \iff x\geq\frac{1}{2}$$

$x=\frac{1}{2}$ punto di MIN. ASS. $f\left(\frac{1}{2}\right)=\ln\left(\frac{1}{4}-\frac{1}{2}+1\right)$

$=\ln\left(\frac{3}{4}\right)<0$.

f decresce in $\left(-\infty,\frac{1}{2}\right)$ e cresce in $\left(\frac{1}{2},+\infty\right)$.

Poiché $\lim_{x\to\pm\infty} f(x)=+\infty$, allora $\nexists$ MAX. ASS.

CONVESSITÀ:

$$f''(x)=\frac{2(x^2-x+1)-(2x-1)^2}{(x^2-x+1)^2}=\frac{2x^2-2x+2-4x^2+4x-1}{(x^2-x+1)^2}$$

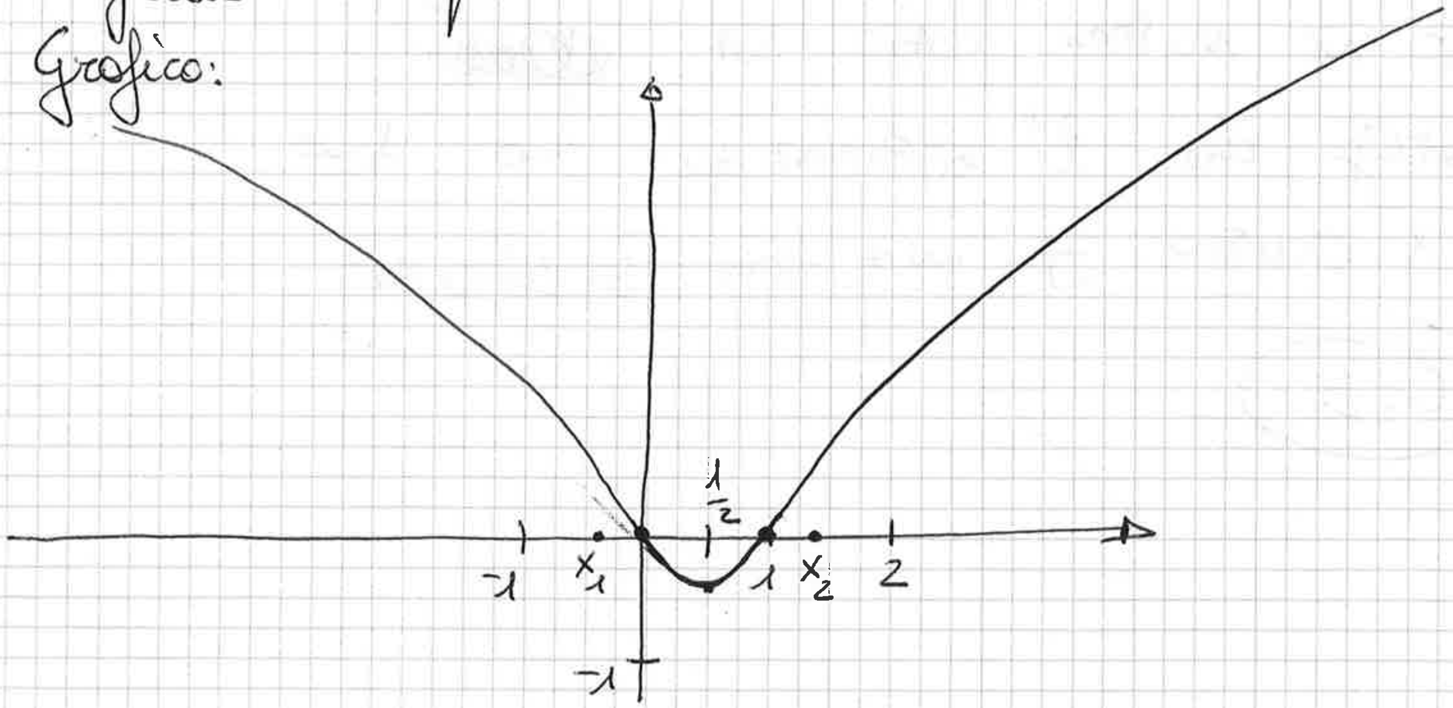
$$= \frac{-2x^2+2x+1}{(x^2-x+1)^2} \geq 0 \iff 2x^2-2x-1 \leq 0$$ C4

$$x_{1,2} = \frac{1 \pm \sqrt{3}}{2} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$$

f concava in $\left(-\infty, \frac{1}{2} - \frac{\sqrt{3}}{2}\right)$; convessa in $\left(\frac{1}{2} - \frac{\sqrt{3}}{2}; \frac{1}{2} + \frac{\sqrt{3}}{2}\right)$; concava in $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}, +\infty\right)$.

~~x₁₂ punti di flesso~~ $x_1 = \frac{1}{2} - \frac{\sqrt{3}}{2}$ punto di flesso obliquo ascendente; $x_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}$ punto di flesso obliquo discendente.

Grafico:



5) $$\frac{2z-2}{z-i} = \frac{(2x-2)+2iy}{x+i(y-1)} \qquad z \neq i$$

$$= \frac{[(2x-2)+2iy][x+i(1-y)]}{x^2+(y-1)^2}$$

~~(2x-2)x+~~

$$\mathrm{Im}\left(\frac{2z-2}{z-i}\right)=\frac{(2x-2)(1-y)+2xy}{x^2+(y-1)^2}=1$$

C5

$$2x-2-\cancel{2xy}+2y+\cancel{2xy}=x^2+y^2-2y+1$$

$$x^2-2x+1+y^2-4y+4-4+2=0$$

$$(x-1)^2+(y-2)^2=2$$

Circonferenza di centro $C\equiv(1,2)$ e di raggio $\sqrt{2}$,

PRIVA DEL PUNTO $z=i$.