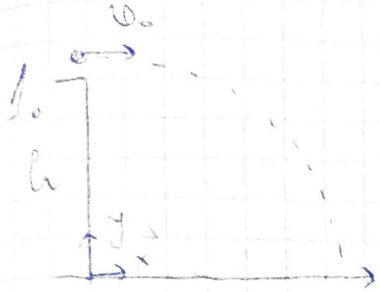
13/06/2024 - Prova Scritta - Soluzioni

1.



$x(t) = v_0 \cdot t$

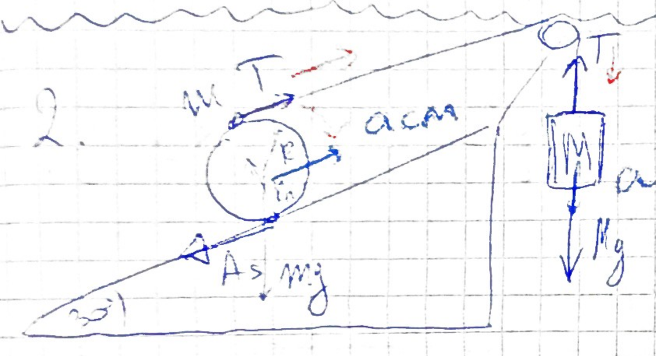
$y(t) = h - g\frac{t^2}{2}$

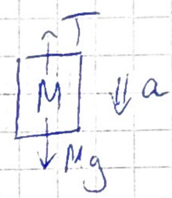
$y(t_g) = 0$

$\Rightarrow$ a) $h - g\frac{t_g^2}{2} = 0 \Rightarrow \boxed{t_g = \sqrt{\frac{2h}{g}} = 5\,s}$

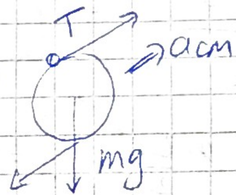
b) $x_g = x(t_g) = v_0 t_g \Rightarrow \boxed{x_g = 50\,m}$

2.

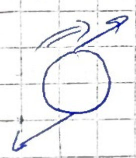




(1) $Ma = Mg - T$

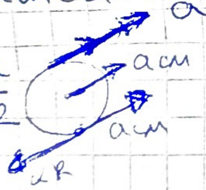


(2) $m a_{cm} = T - A_s - mg\sin\theta$ $\quad$ "1/2"



(3) $I\alpha = (T + A_s)R$

Puro rotolamento: $a = 2a_{cm}$



$\alpha = \frac{a_{cm}}{R} = \frac{a}{2R}$

$\Rightarrow$ (3) $\frac{m\cancel{R^2}}{2}\cdot\frac{a_{cm}}{\cancel{R}} = (T + A_s)\cdot\cancel{R} \Rightarrow \frac{m a_{cm}}{2} = T + A_s$

(2)+(3): $m a_{cm} + \frac{m a_{cm}}{2} = T - \cancel{A_s} - mg\sin\theta + T + \cancel{A_s}$

$\Rightarrow \frac{3}{2} m a_{cm} = 2T - \frac{mg}{2}$

(1)·2 $\quad 2\cdot M\cdot 2a_{cm} = 2Mg - 2T$

$\oplus \quad a_{cm}\left(\frac{3}{2}m + 4M\right) = 2Mg - \frac{mg}{2}$

$a_{cm}\left(\frac{3\cancel{m}}{2} + 4\cdot\frac{3\cancel{m}}{2}\right) = g\left(2\cdot 4\frac{\cancel{m}}{2} - \frac{\cancel{m}}{2}\right) \Rightarrow \boxed{a_{cm} = g\cdot\frac{11}{27}}$

3.

$$m v_0 \cdot \frac{L}{2} = I_{tot} \cdot \omega$$

$$I_{tot} = I_{sb} + I_m = \frac{ML^2}{12} + m\left(\frac{L}{2}\right)^2 =$$

$$= \frac{9mL^2}{12} + \frac{mL^2}{4} \cdot \frac{3}{3} = mL^2$$

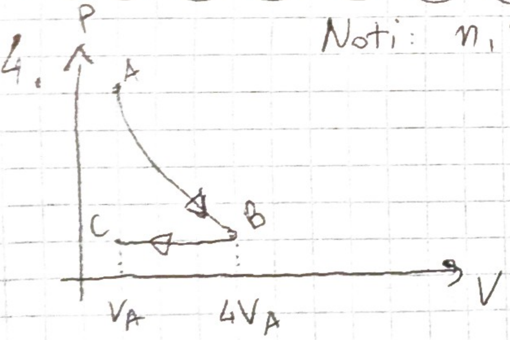
a) $$\boxed{\omega =} \frac{\cancel{m} v_0 \cdot \cancel{L}/2}{\cancel{m} L^{\cancel{2}}} = \boxed{\frac{v_0}{2L} = 15 \, \frac{rad}{s}}$$

b) $$E_{persa} = E_{prima} - E_{dopo} = \frac{m v_0^2}{2} - \frac{I\omega^2}{2} = \frac{m v_0^2}{2} - \frac{m\cancel{L^2}}{2} \cdot \frac{v_0^2}{4\cancel{L^2}} = \frac{m v_0^2}{2}\left(\frac{3}{4}\right)$$

$$\frac{E_{persa}}{E_{prima}} = \frac{\cancel{\frac{m v_0^2}{2}} \cdot \frac{3}{4}}{\cancel{\frac{m v_0^2}{2}}} = \boxed{75\%}$$

c) $$E_{persa} = \cancel{m} \cdot c \cdot \Delta T = \frac{3}{4} \frac{\cancel{m} v_0^2}{2} \Rightarrow \boxed{\Delta T = \frac{3}{4} \cdot \frac{v_0^2}{2c} = 0.75 \, °C}$$

4. Noti: n, T_A, R, mono at.



	P	V	T
A	p_A	V_A	T_A
B	$p_A 4^{-\gamma}$	$4V_A$	$T_A \cdot 4^{1-\gamma}$
C	$p_A 4^{-\gamma}$	V_A	$T_A 4^{-\gamma}$

$$p_A V_A^{\gamma} = p_B V_B^{\gamma}$$
$$\Downarrow$$
$$p_B = p_A \cdot \left(\frac{V_A}{V_B}\right)^{\gamma} = p_A 4^{-\gamma}$$

$$\boxed{\Delta S_{ABC}} = \underset{p=cost}{\Delta S_{BC}} = \int_B^C \frac{dQ}{T} = \int_B^C n c_p \frac{dT}{T} = \boxed{n c_p \ln \frac{T_C}{T_B} = -\frac{5}{2} n R \ln 4}$$

$$\boxed{Q_{ABC}} = \cancel{Q_{AB}}^{\,0} + Q_{BC} = n c_p \Delta T_{BC} = n c_p (T_C - T_B) = \boxed{\frac{5}{2} n R T_A \left(4^{-\gamma} - 4^{1-\gamma}\right)}$$

Em