1 Luglio 2024 - Fisica 1 - Compito Scritto

Soluzioni

1. $\alpha = At$, $A = 1\,\frac{rad}{s^3}$. $R = D$, $t = T \longrightarrow |\vec{a}|_{t=T} = ?$ $\Big\}$ $|\vec{a}| = \sqrt{a_n^2 + a_\tau^2}$

$a_\tau = \alpha \cdot R$, $a_n = \frac{v^2}{R}$, $R = D$

$a_\tau = \frac{dv}{dt}$

$a_\tau = \alpha \cdot R = A \cdot D \cdot t \Rightarrow \frac{dv}{dt} = A \cdot D \cdot t \Rightarrow v(t) = \cancel{v(0)}^{0} + \int_0^t ADt\,dt = AD\frac{t^2}{2}$

$\Rightarrow a_n(t) = \frac{v^2(t)}{D} = \frac{A^2D^{\cancel{2}}t^4}{4\cancel{D}} = \frac{A^2Dt^4}{4}$ $\left[\frac{rad^2}{s^6} \cdot m \cdot s^4 = \frac{m}{s^2} \text{ O.K.}\right]$

$a_\tau(t) = ADt$ $\left[\frac{rad}{s^3} \cdot m \cdot s = \frac{m}{s^2} \text{ O.K.}\right]$

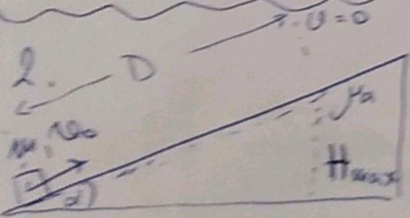
$a_n(T) = \frac{A^2DT^4}{4}$

$a_\tau(T) = ADT$

$a(T) = \sqrt{\frac{A^4D^2T^8}{16} + A^2D^2T^2} = \sqrt{A^2D^2T^2\left(1 + \frac{A^2T^6}{16}\right)} =$

$$\boxed{a(T) = ADT\sqrt{1 + \frac{A^2T^6}{16}}}$$ $\left[\frac{rad}{s^3} \cdot m \cdot s \cdot \sqrt{1 + \frac{rad^2}{s^6}s^6} = \frac{m}{s^2} \text{ OK}\right]$

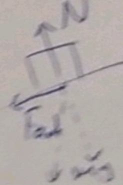
2.



$E_f - E_i = W^{\text{forze non con.}}$

$E_f = mgH_{max}$

$E_i = \frac{mv_0^2}{2}$



$F_{ad} = \mu_d \cdot N = \mu_d mg\cos\alpha$

$W^{\text{forze non cons.}} = W_{\text{attrito din.}} = -|\vec{F}_{ad}|D = -\mu_d mg\cos\alpha \cdot D$

$\sin\alpha = \frac{H_{max}}{D}$

$\Rightarrow H_{max} = D \cdot \sin\alpha \Rightarrow \cancel{m}gD\sin\alpha - \frac{\cancel{m}v_0^2}{2} = -\mu_d\cancel{m}g\cos\alpha \cdot D$

$\Rightarrow D(g\sin\alpha + \mu_d g\cos\alpha) = \frac{v_0^2}{2} \Rightarrow \boxed{D = \frac{v_0^2}{2g}\frac{1}{\sin\alpha + \mu_d\cos\alpha}}$ $\left[\frac{\frac{m^2}{s^2}}{\frac{m}{s^2}} = m \text{ OK}\right]$

$H_{max} = D\sin\alpha = \frac{v_0^2}{2g}\frac{\sin\alpha}{\sin\alpha + \mu_d\cos\alpha} = \frac{v_0^2}{2g}\frac{1}{1 + \mu_d ctg(\alpha)}$

$W_{\text{attrito din.}} = -\mu_d mg\cos\alpha \cdot D = -\mu_d m\cancel{g}\cos\alpha\frac{v_0^2}{2\cancel{g}}\frac{1}{\sin\alpha + \mu_d\cos\alpha} = \boxed{\frac{-\mu_d m v_0^2}{tg\alpha + \mu_d}}$ $\left[mv_0^2 = J \text{ OK}\right]$

3.

piccole osc. $\approx \theta$

$$I_{tot}\cdot\frac{d^2\theta}{dt^2} = \sum M = -m_{tot}\cdot g\cdot d_{CM-O}\sin\theta$$

$$d_{CM-O} = \frac{2m\cdot L/4 + m\cdot 3L/4 - m\cdot L/4}{4m} = \frac{4mL/4}{4m} = L/4$$

$$I_{tot} = I_{asta} + m\cdot\left(\frac{L}{4}\right)^2 + m\cdot\left(\frac{3L}{4}\right)^2$$

$$I_{asta} = 2m\cdot\frac{L^2}{12} + 2m\cdot\left(\frac{L}{4}\right)^2 = \frac{2mL^2\cdot 4}{12\cdot 4} + \frac{2mL^2\cdot 3}{16\cdot 3} = \frac{14mL^2}{48}$$

$$I_{tot} = \frac{mL^2\cdot 3}{16\cdot 3} + \frac{9mL^2\cdot 3}{16\cdot 3} + \frac{14mL^2}{48} = \frac{3mL^2+27mL^2+14mL^2}{48} = \frac{44mL^2}{48} = \frac{11mL^2}{12}$$

$$I_{tot}\cdot\frac{d^2\theta}{dt^2} + \cancel{4}mg\cdot\frac{L}{\cancel{4}}\cdot\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \overbrace{\frac{mgL}{11mL^2}\cdot 12}^{\Omega_0^2}\cdot\theta = 0$$

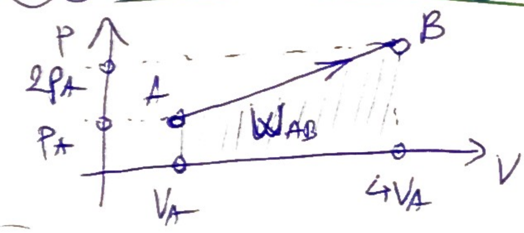
$$\Omega_0^2 = \frac{12\,mgL}{11\,mL^2} = \frac{12}{11}\cdot\frac{g}{L} \Rightarrow \boxed{T = \frac{2\pi}{\Omega_0} = 2\pi\sqrt{\frac{11L}{12g}}} \quad \left[\sqrt{\frac{m}{m}s^2} = s \quad O.K.\right]$$

4.

$$p = \frac{p_A}{3}\left(2+\frac{V}{V_A}\right)$$

$V_B = 4V_A$

$\Downarrow$



$$T_A = \frac{p_AV_A}{nR} \qquad T_B = \frac{2p_A 4V_A}{nR} = 8T_A$$

$$p_B = p(V=V_B) = \frac{p_A}{3}\left(2+\frac{4V_A}{V_A}\right) = 2p_A$$

$$W_{AB} = \int_A^B p\,dV = \int_A^B \frac{p_A}{3}\left(2+\frac{V}{V_A}\right)dV = \frac{p_A}{3}\left(2\int_A^B dV + \frac{1}{V_A}\int_A^B V\,dV\right) = \frac{p_A}{3}\left(2(4V_A-V_A)+\frac{1}{V_A}\left(\frac{16V_A^2-V_A^2}{2}\right)\right) =$$

$$= \frac{p_A}{3}\left(2\cdot 3V_A + \frac{1}{2V_A}15V_A^2\right) = \frac{p_A}{3}\left(\frac{6V_A\cdot 2}{2} + \frac{15V_A}{2}\right) = \frac{p_A}{3}\cdot\frac{27V_A}{2} = \frac{9p_AV_A}{2}$$

$$\Delta S_{AB} = \int_A^B \frac{\delta Q}{T} = \int_A^B \frac{p\,dV + nc_v\,dT}{T} = \int_A \frac{nRT\,dV}{V\cdot T} + \int_A^B \frac{nc_v\,dT}{T} = nR\ln\frac{V_B}{V_A} + nc_v\ln\frac{T_B}{T_A}$$

$$\Rightarrow \boxed{\Delta S_{AB} = nR\ln 4 + nc_v\ln 8}$$

$$\Delta S_{AB} = 2nR\ln 2 + 3\,n\frac{3}{2}R\ln 2 = nR\ln 2\left(2+\frac{9}{2}\right) = nR\ln 2\cdot\frac{13}{2}$$