ABSTRACT: We consider the open unit disk $\mathbb{D}$ equipped with the hyperbolic metric and the associated hyperbolic Laplacian $\mathcal{L}$. For $\lambda \in \mathbb{C}$ and $n \in \mathbb{N}$, a λ-polyharmonic function of order n is a function $f : \mathbb{D} \to \mathbb{C}$ such that $(\mathcal{L} - \lambda I)^n f = 0$. If $n = 1$, one gets λ-harmonic functions. Based on a Theorem of Helgason on the latter functions, we prove a boundary integral representation theorem for λ-polyharmonic functions. For this purpose, we first determine n^{th}-order λ-Poisson kernels. Subsequently, we introduce the λ-polyspherical functions and determine their asymptotics at the boundary $\partial\mathbb{D}$, i.e., the unit circle. In particular, this proves that, for eigenvalues not in the interior of the L^2-spectrum, the zeroes of these functions do not accumulate at the boundary circle. Hence the polyspherical functions can be used to normalise the n^{th}-order Poisson kernels. By this tool, we extend to this setting several classical results of potential theory: namely, we study the boundary behaviour of λ-polyharmonic functions, starting with Dirichlet and Riquier type problems and then proceeding to Fatou type admissible boundary limits.

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