

Laboratorio di fisica sperimentale

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meccanica ingegneria



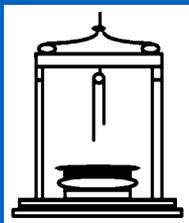
<https://corsidilaurea.uniroma1.it/it/users/marcotoppiuniroma1i>

# LABORATORIO DI FISICA SPERIMENTALE

Ingegneria meccanica



A.A. 2023-2024



## Quinta esperienza: il volano

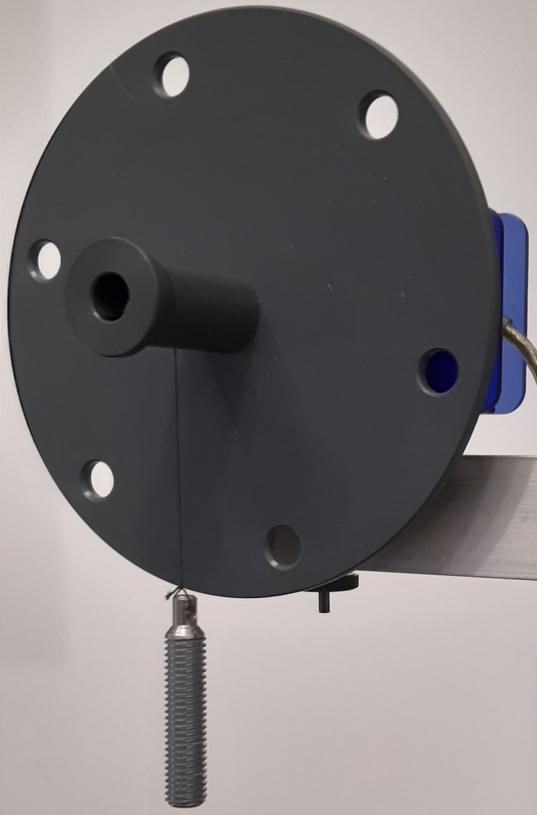
lasciate il tavolo di laboratorio in ordine e pulito;  
ne siete responsabili (anche della strumentazione)



PERNO

VITE

DADO



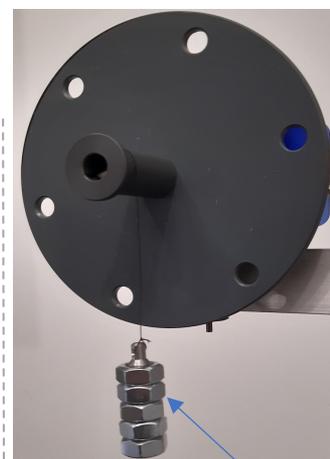
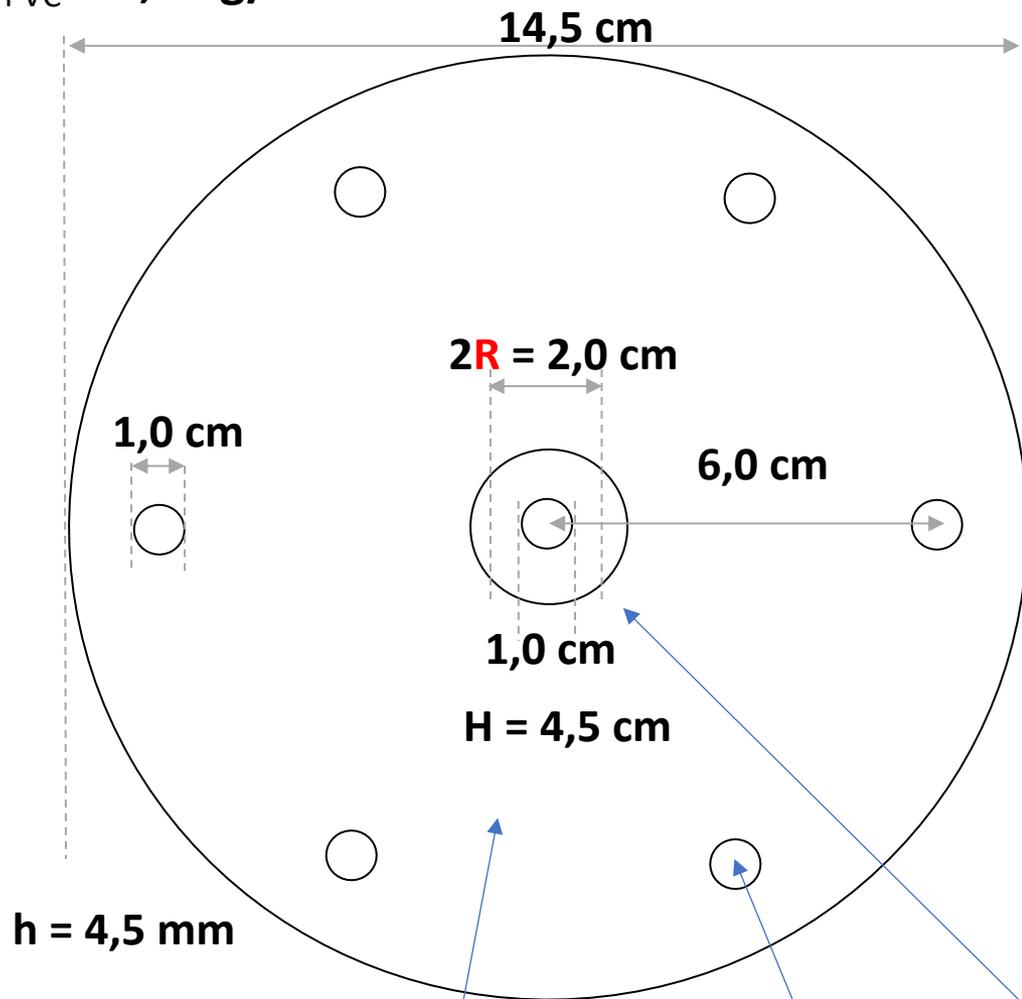
vite + dado = bullone  
→ momento d'inerzia



dado → massa

VOLANO: organo rotante costruito in modo  
da presentare un elevato *momento di inerzia*  
[Treccani]

$$\rho_{\text{PVC}} = 1,42 \text{ g/cm}^3$$



**MASSA DADO ( $\Delta m$ ) 10 g**

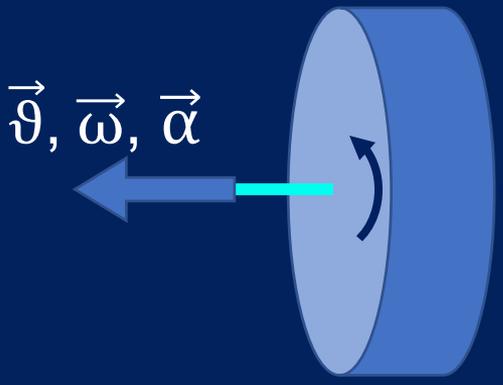
**MASSA VITE 22 g**

**MASSA BULLONE 32 g**

**MASSA PERNO ( $m_0$ ) 25 g**

$$I_0 = [2770 \text{ (disco)} - 50 \text{ (6 fori)} + 10 \text{ (perno)}] = 2730 \text{ g cm}^2$$

$$\Delta I = 1150 \text{ (bullone)} \text{ g cm}^2$$



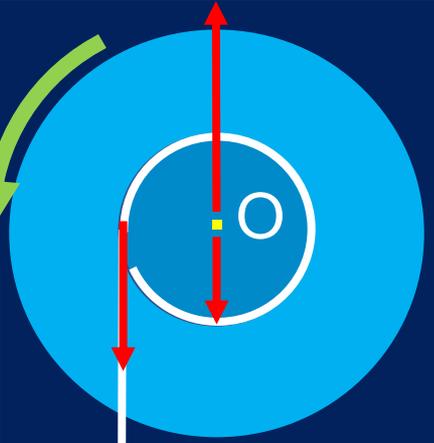
$\vec{\vartheta}, \vec{\omega}, \vec{\alpha}$

momenti delle forze esterne

$$\sum_{i=1,N} \vec{M}_i^{(e)} = I \vec{\alpha}$$

momento d'inerzia assiale

accelerazione angolare



+

I e II cardinale della dinamica



$$\vec{M}_{PV} + \vec{M}_{RV} + \vec{M}_T + \vec{M}_{att} = I \vec{\alpha}$$

$$0 + 0 + R(mg - m\alpha R) - M_{att} = I\alpha$$

$$Rmg - M_{att} = I\alpha + m\alpha R^2$$

$$\alpha = \frac{Rmg - M_{att}}{I + mR^2}$$

$$\vec{T} + m \vec{g} = m \vec{a}$$

$$-T + mg = m\alpha R$$

$$T = mg - m\alpha R$$

$$\alpha_{\text{discesa}} = \frac{Rmg - M_{\text{att}}}{I + mR^2}$$

$$\alpha_{\text{salita}} = \frac{Rmg + M_{\text{att}}}{I + mR^2}$$

$$\alpha = \frac{\alpha_{\text{salita}} + \alpha_{\text{discesa}}}{2} = \frac{Rmg}{I + mR^2}$$

$$\alpha_{\text{attrito}} = \frac{\alpha_{\text{salita}} - \alpha_{\text{discesa}}}{2} = \frac{M_{\text{att}}}{I + mR^2}$$

$$\alpha = \frac{Rm_0g}{I_0 + N\Delta I + m_0R^2}$$

$$\frac{1}{\alpha} = \frac{I_0 + N\Delta I + m_0R^2}{Rm_0g}$$

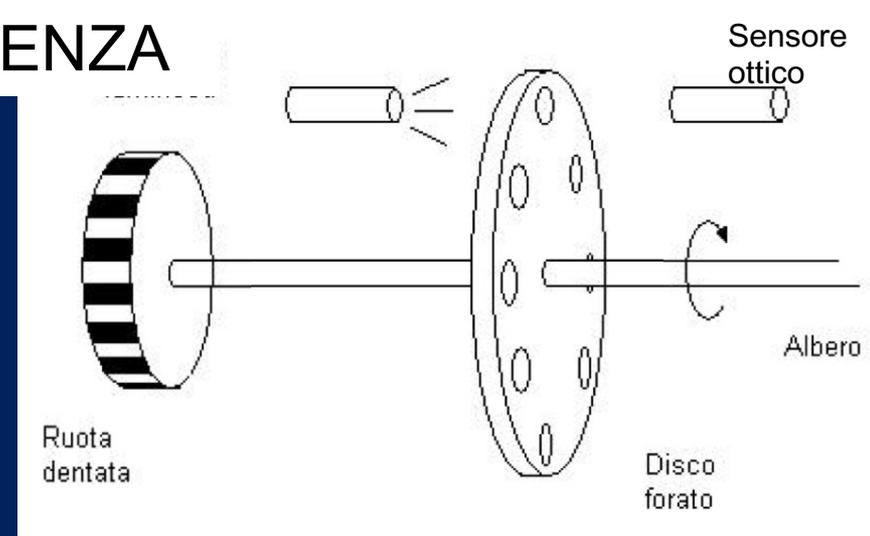
$$\frac{1}{\alpha} = N \frac{\Delta I}{Rm_0g} + \frac{I_0 + m_0R^2}{Rm_0g}$$



$$\begin{aligned} I &= I_0 + N \Delta I \\ m &= m_0 \end{aligned}$$

RICAVARE  $I_0$  DA q

# I SENSORI PER QUESTA ESPERIENZA



**sensore angolare**

4000 punti:  $360^\circ/4000 = 0,09^\circ = 1,57 \text{ mrad}$

|                       |   |
|-----------------------|---|
| Three-step Pulley     | 10, 29 and 48 mm diameter   |
| Sensor Dimensions     | 10 cm by 5 cm by 3.75 cm, 6.35 mm diameter shaft  |
| Resolution            | $\pm 0.09^\circ$ or 0.0078 mm<br>0.02 mm (linear) and $0.09^\circ$ (angular) at 4,000 points per revolution |
| Rotational Resolution | 0.00157 radian  |
| Maximum Rotation Rate | 30 revolutions per second   |
| Optical Encoder       | Bidirectional, indicates direction of motion, 4,000 divisions/revolution                                    |

massima velocità angolare misurabile:  $2\pi \times 30 \text{ Hz} = 200 \text{ rad/s}$

PROMEMORIA:

nome, cognome, FIRMA

accelerazioni angolari con Capstone

dati → foglio

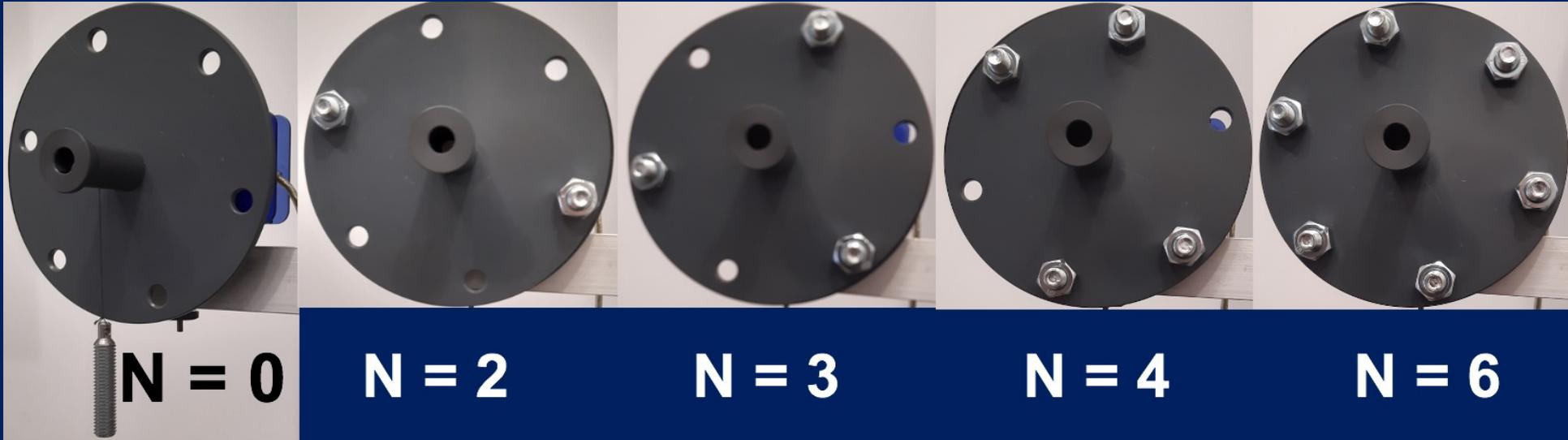
grafici (con excel)

valutazione di  $q$  e di  $p$  → (con LabCalc)

conclusioni → foglio

- 1) studio del moto al variare del momento di inerzia**
- 2) studio del moto al variare della massa**





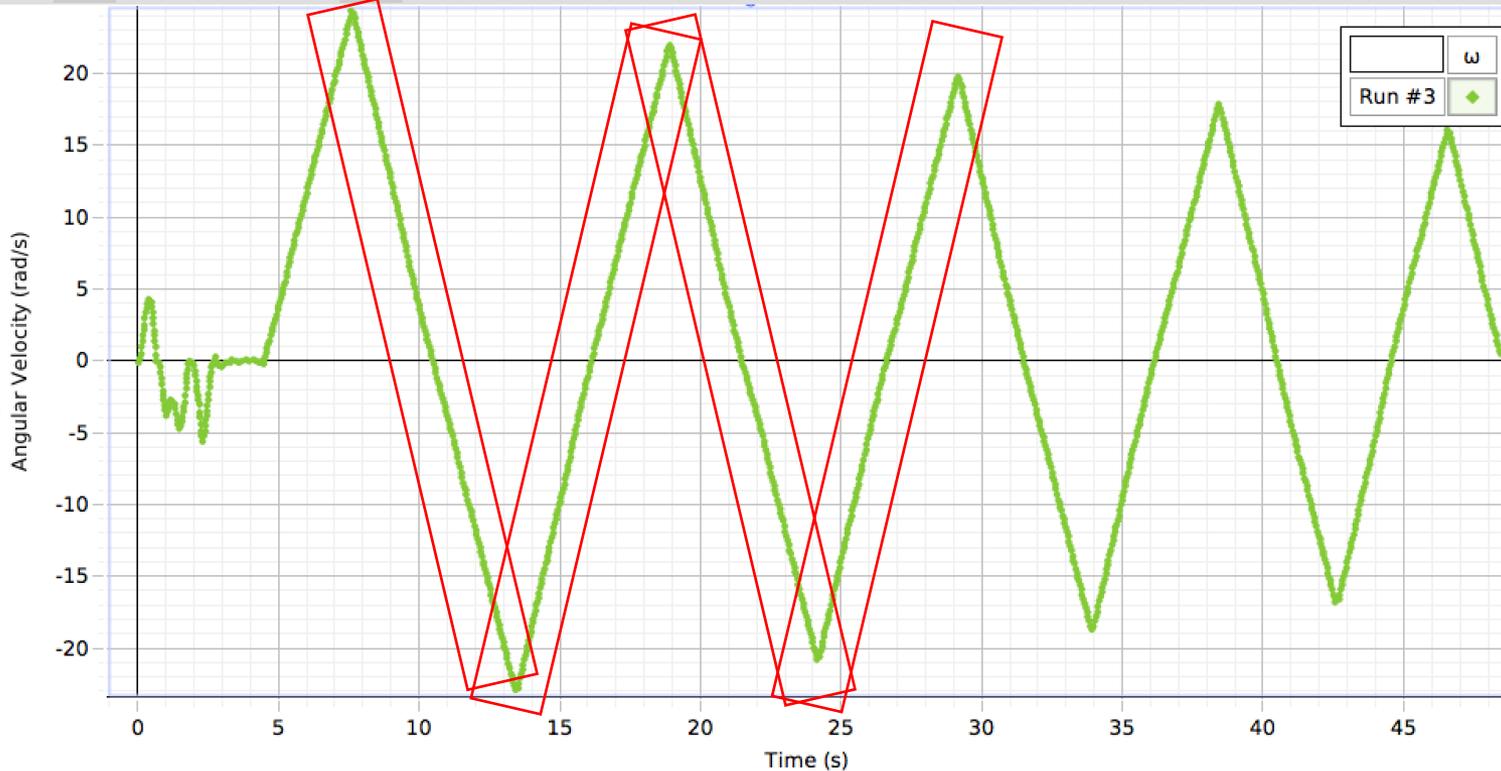
$$I = I_0 + N \Delta I$$

2730 g cm<sup>2</sup>

1150 g cm<sup>2</sup>



**Angular velocity (rad/s)**



[Graph title here]

Record Continuous Mode 00:48,47 Ready Rotary Motion Sensor 100,00 Hz Recording Conditions Delete Last Run

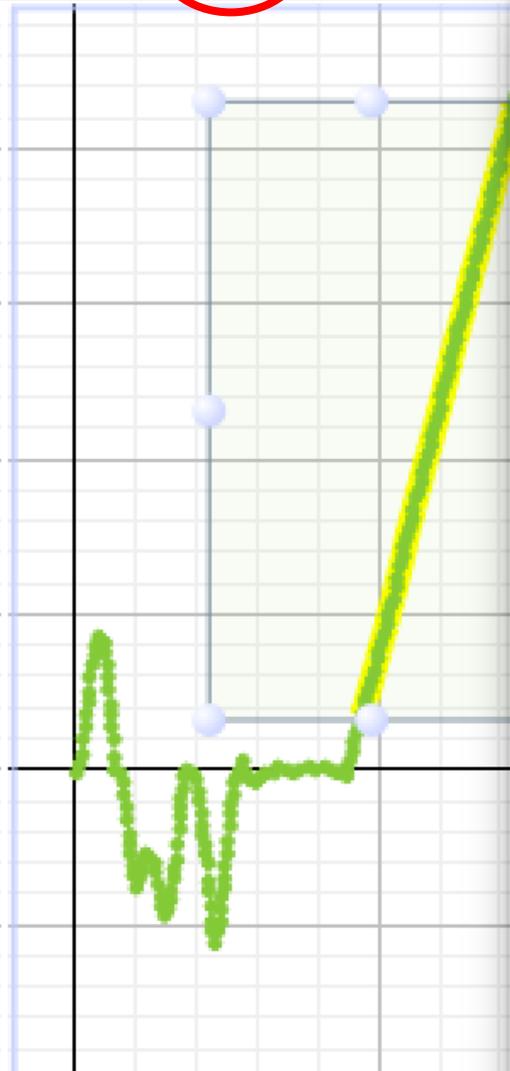
**Record**

**100,00 Hz**

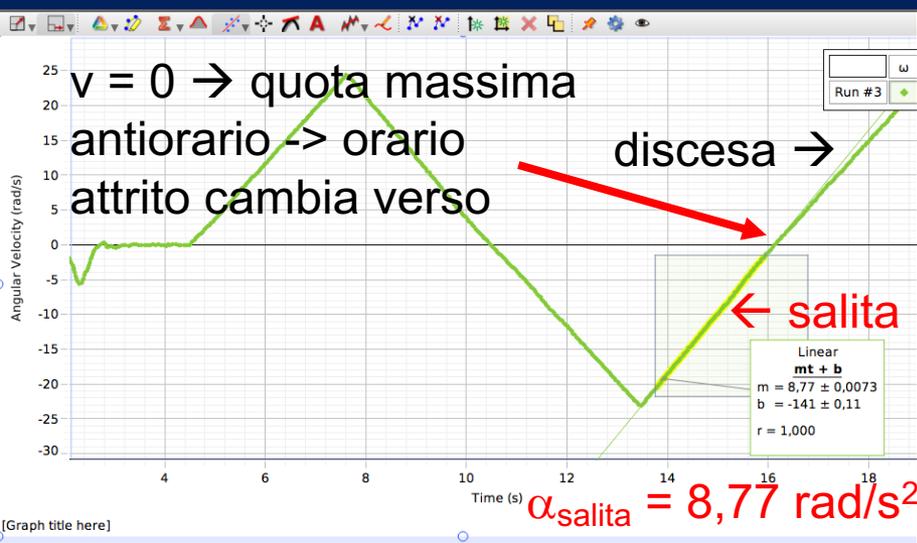
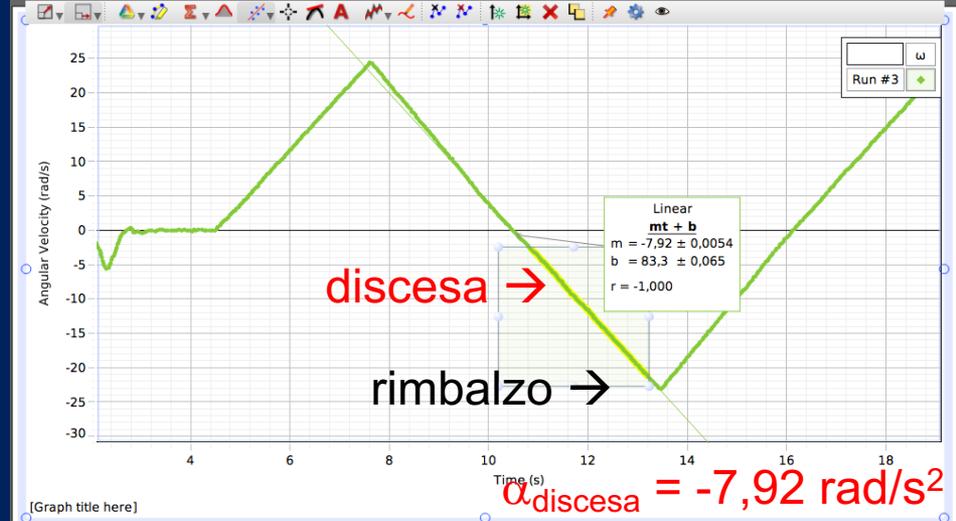
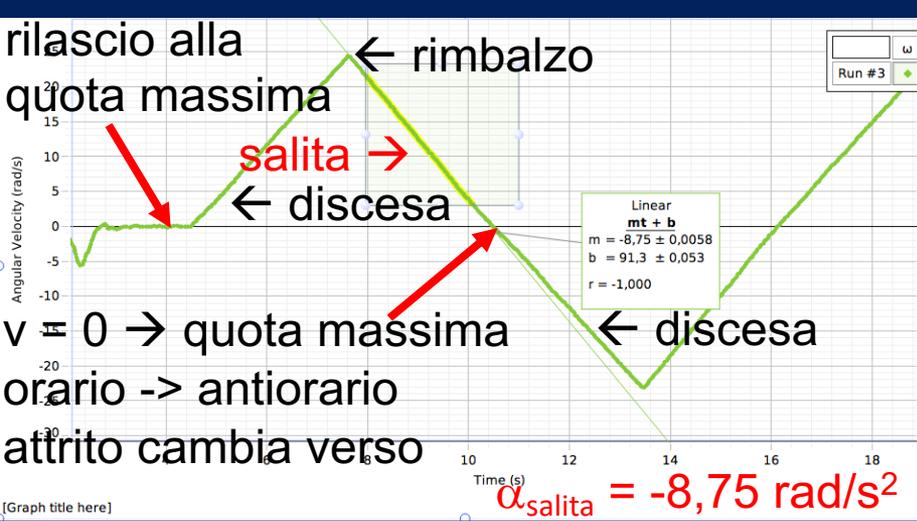
**ACQUISIRE 3-4 TRATTI SALITA-DISCESA PER OGNI MOMENTO D'INERZIA (5)**

Angular Velocity (rad/s)

20  
15  
10  
5  
0  
-5  
-10



- Proportional:  $At$
- Linear:  $mt + b$
- Weighted Linear:  $mt + b$
- Quadratic:  $At^2 + Bt + C$
- Cubic:  $A + Bt + Ct^2 + Dt^3$
- Polynomial:  $A + Bt + Ct^2 + \dots + C$
- Power:  $A(t-t_0)^n + B$
- Inverse (no offset):  $A/t + B$
- Inverse:  $A/(t-t_0) + B$
- Inverse Square (no offset):  $A/t^2 + B$
- Inverse Square:  $A/(t-t_0)^2 + B$
- Inverse Power:  $A/(t-t_0)^n + B$
- Natural Exponential:  $Ae^{(-Bt)}$
- Natural Logarithm:  $A \ln(B(t-t_0))$
- Base-10 Exponential:  $A 10^{(Bt)}$
- Base-10 Logarithm:  $A \log(B(t-t_0))$
- Inverse Exponential:  $A/(1 - e^{(Bt)})$



$$\alpha_{salita} = \frac{Rmg + M_{att}}{I + mR^2}$$

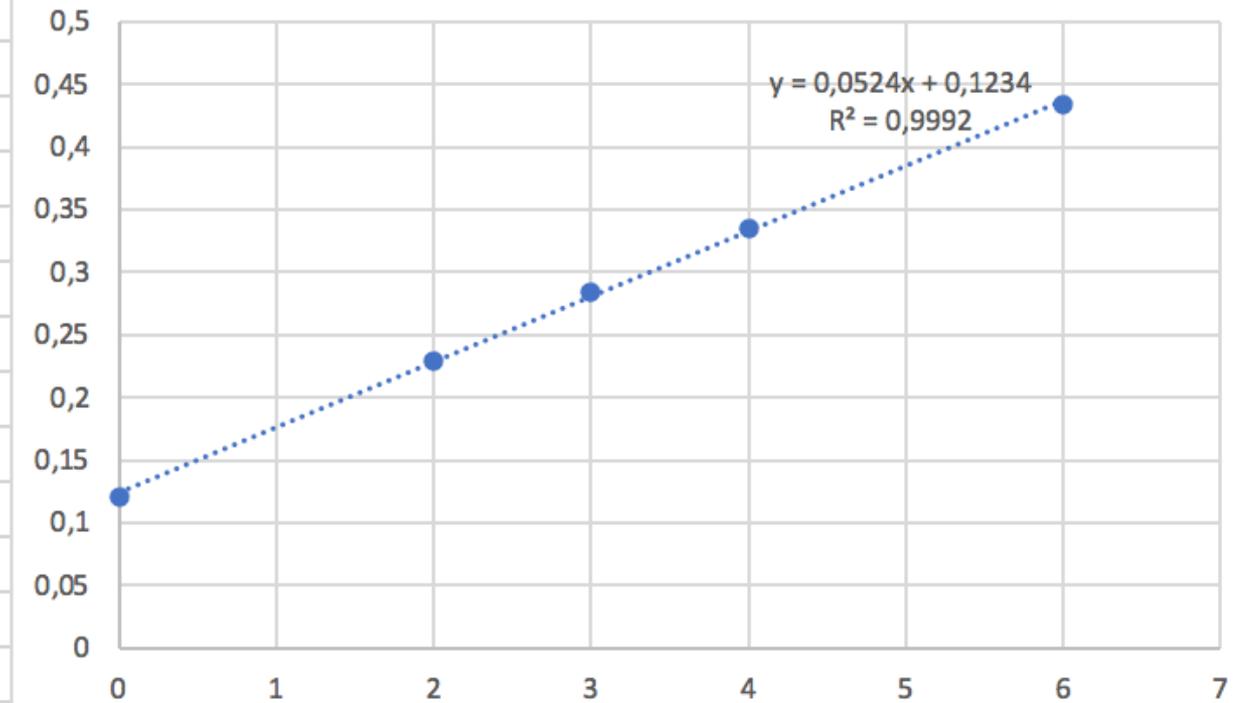
$$\alpha_{discesa} = \frac{Rmg - M_{att}}{I + mR^2}$$

|    | O          | P     | Q                 | R | S       | T     | U    | V |
|----|------------|-------|-------------------|---|---------|-------|------|---|
| 12 |            |       |                   |   |         |       |      |   |
| 13 |            |       |                   |   |         |       |      |   |
| 14 |            | 7,92  |                   |   |         | 7,92  |      |   |
| 15 |            | -8,73 |                   |   |         | -8,73 |      |   |
| 16 | ORARIO     | 8,86  | =ASS((P16+P17)/2) |   | SALITA  | 8,86  | 8,43 |   |
| 17 |            | 7,99  |                   |   | DISCESA | 7,99  |      |   |
| 18 | ANTIORARIO | -8,74 | 8,36              |   | SALITA  | -8,74 | 8,36 |   |
| 19 |            | -7,98 |                   |   | DISCESA | -7,98 |      |   |
| 20 | ORARIO     | 8,97  | 8,47              |   | SALITA  | 8,97  | 8,47 |   |
| 21 |            | 7,96  |                   |   | DISCESA | 7,96  |      |   |
| 22 | ANTIORARIO | -8,60 | 8,22              |   | SALITA  | -8,60 | 8,22 |   |
| 23 |            | -7,83 |                   |   | DISCESA | -7,83 |      |   |
| 24 | ORARIO     | 8,58  | 8,19              |   | SALITA  | 8,58  | 8,19 |   |
| 25 |            | 7,80  |                   |   | DISCESA | 7,80  |      |   |
| 26 |            | -8,62 |                   |   |         | -8,62 |      |   |
| 27 |            |       | =MEDIA(Q16:Q24)   |   |         |       | 8,33 |   |

ACCELERAZIONE  
ANGOLARE ( $I = I_0$ )

SOLO 3-4 TRATTI SALITA-DISCESA

| alfa  | N | 1/alfa |
|-------|---|--------|
| 8,331 | 0 | 0,120  |
| 4,358 | 2 | 0,229  |
| 3,514 | 3 | 0,285  |
| 2,985 | 4 | 0,335  |
| 2,303 | 6 | 0,434  |



$$\frac{1}{\alpha} = N \frac{\Delta I}{Rm_0g} + \frac{I_0 + m_0R^2}{Rm_0g}$$

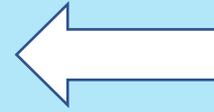
RICAVARE  $I_0$  DA q

$$m_0 = 25 \text{ g}$$

$$R = 1 \text{ cm}$$

**CALCOLARE LO SCARTO RELATIVO** DA  $I_0^* = 2730 \text{ g cm}^2$

- 1) studio del moto al variare del momento di inerzia**
- 2) studio del moto al variare della massa**

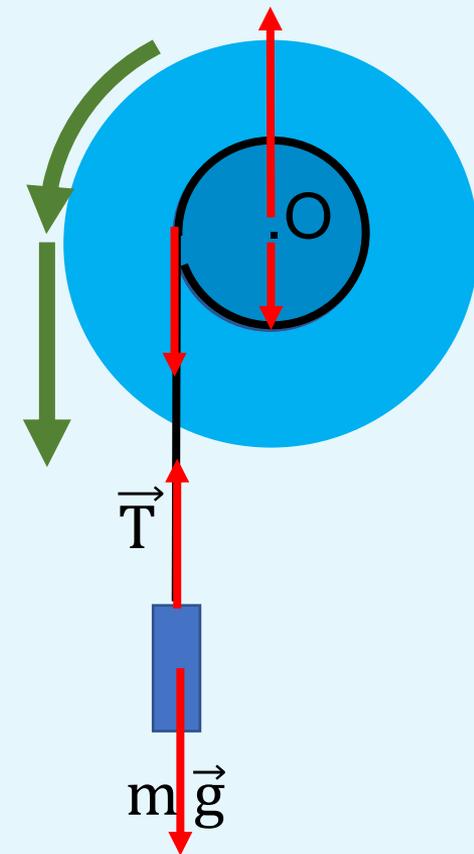


$$\alpha = \frac{\alpha_{\text{salita}} + \alpha_{\text{discesa}}}{2} = \frac{Rmg}{I + mR^2}$$

$$\alpha_{\text{attrito}} = \frac{\alpha_{\text{salita}} - \alpha_{\text{discesa}}}{2} = \frac{M_{\text{att}}}{I + mR^2}$$

$$\alpha = \frac{Rm_0g + N R\Delta mg}{I_0 + m_0R^2 + N \Delta mR^2} \approx \frac{Rm_0g + N R\Delta mg}{I_0 + m_0R^2}$$

$$\alpha = N \frac{R\Delta mg}{I_0 + m_0R^2} + \frac{Rm_0g}{I_0 + m_0R^2}$$



**RIPORTARE** I VALORI DI  $p$  E  $q$  E IL COEFFICIENTE DI CORRELAZIONE  $R^2$

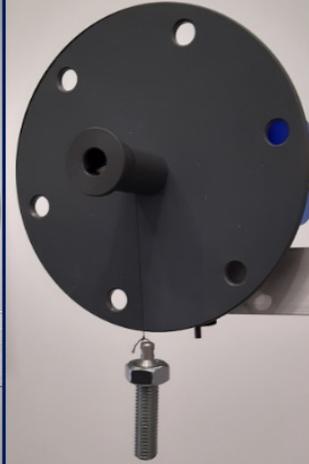
$$I = I_0 \text{ (0 BULLONI)}$$

$$m = m_0 + N \Delta m$$

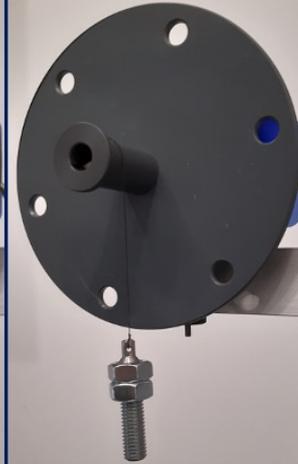
PER VALUTARE SE L'APPROSSIMAZIONE E' VALIDA ( $\alpha_{\text{attrito}}$  E' INDIPENDENTE DA  $N$ ?)  
**VERIFICARE** (LabCalc) SE  $|p| < \sigma_p$



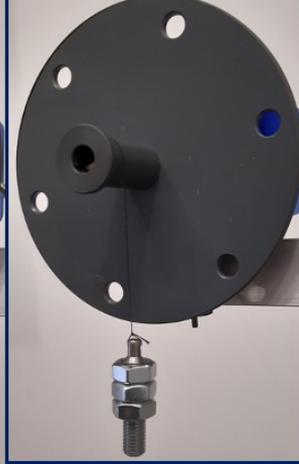
**N = 0**



**N = 1**



**N = 2**



**N = 3**



**N = 4**



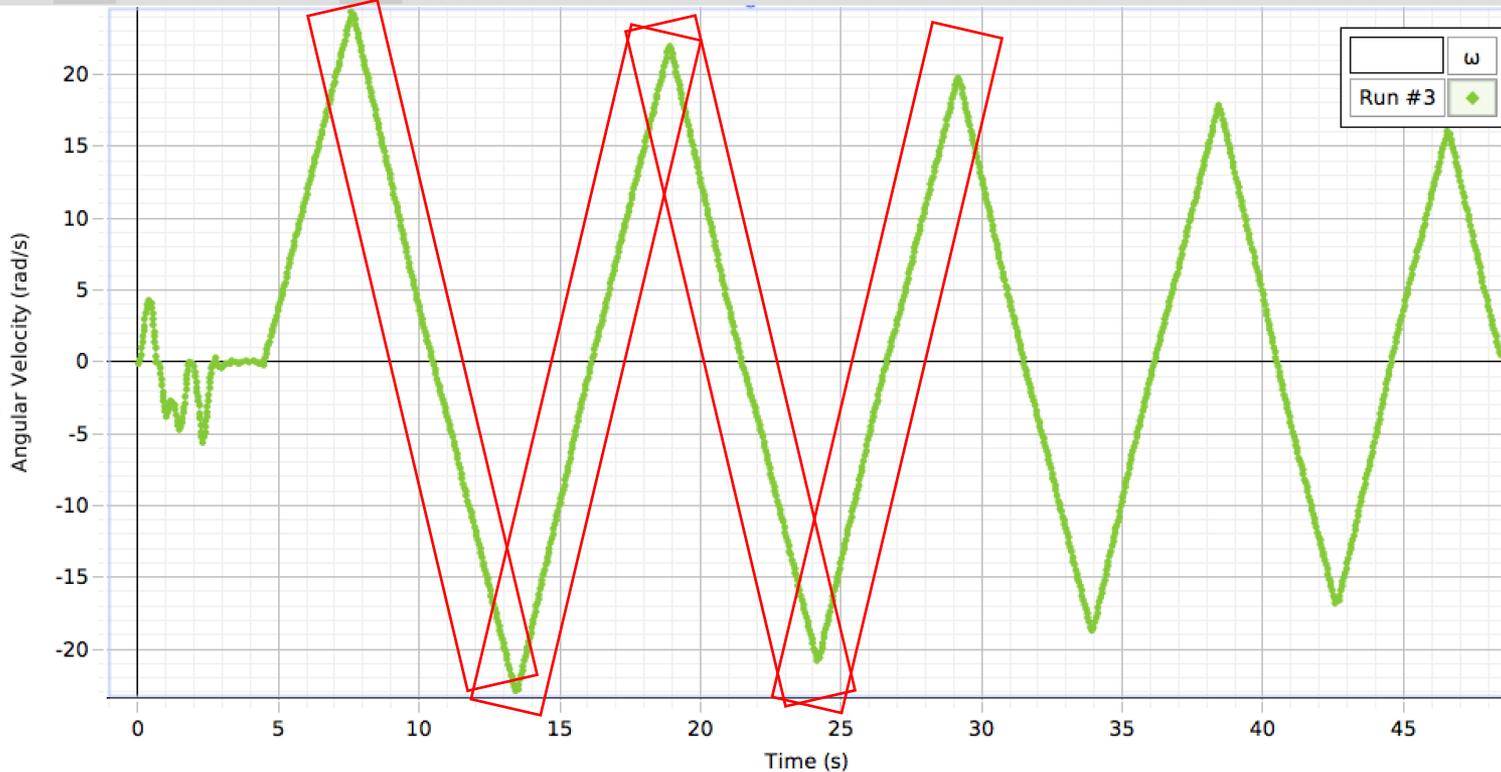
**N = 5**

$$m = m_0 + N \Delta m$$

↑  
**25 g**

↑  
**10 g**

Angular velocity (rad/s)



Record

100,00 Hz

AQUISIRE 3-4 TRATTI SALITA-DISCESA  
PER OGNI MASSA DEL PESETTO (6)

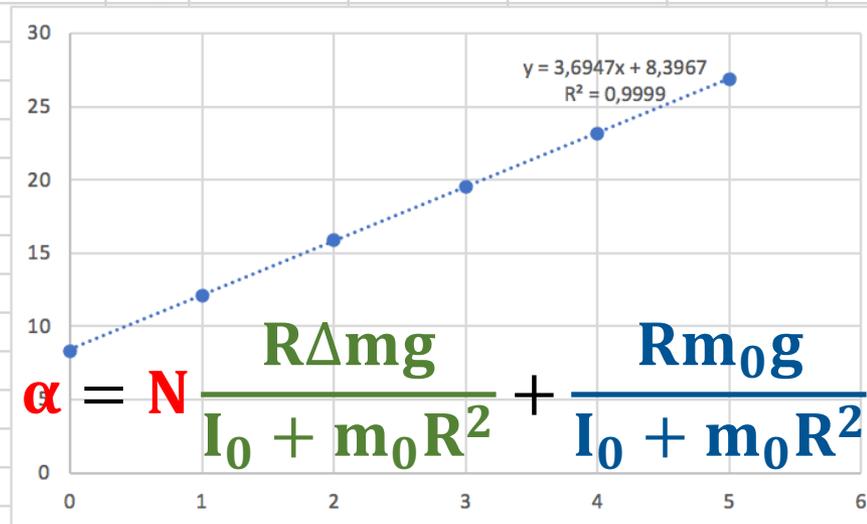
# SEMISOMME SALITA/DISCESA: CONTRIBUTO DELL'INERZIA DEL DISCO

| N=0   |      | N=1   |       | N=2   |       | N=3   |       | N=4   |       | N=5   |       |
|-------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 8,86  | 8,43 | 12,6  | 12,15 | -16,3 | 15,85 | -20   | 19,55 | -23,6 | 23,15 | 27,5  | 27,00 |
| 7,99  |      | 11,7  |       | -15,4 |       | -19,1 |       | -22,7 |       | 26,5  |       |
| -8,74 | 8,36 | -12,5 | 12,05 | 16,3  | 15,85 | 20,1  | 19,65 | 23,6  | 23,15 | -27   | 26,60 |
| -7,98 |      | -11,6 |       | 15,4  |       | 19,2  |       | 22,7  |       | -26,2 |       |
| 8,97  | 8,47 | 12,6  | 12,10 | -16,4 | 15,95 | -20   | 19,55 | -23,6 | 23,15 | 27,5  | 27,00 |
| 7,96  |      | 11,6  |       | -15,5 |       | -19,1 |       | -22,7 |       | 26,5  |       |
| -8,60 | 8,22 | -12,5 | 12,05 | 16,5  | 16,00 | 20    | 19,55 | 23,6  | 23,20 | -27,2 | 26,70 |
| -7,83 |      | -11,6 |       | 15,5  |       | 19,1  |       | 22,8  |       | -26,2 |       |
| 8,58  | 8,19 | 12,5  | 12,05 | -16,2 | 15,75 | -19,9 | 19,45 | -23,4 | 23,00 | 27,2  | 26,85 |
| 7,80  |      | 11,6  |       | -15,3 |       | -19   |       | -22,6 |       | 26,5  |       |

| N | alfa |
|---|------|
| 0 | 8,33 |
| 1 | 12,1 |
| 2 | 15,9 |
| 3 | 19,6 |
| 4 | 23,1 |
| 5 | 26,8 |

$\begin{matrix} 8,33 \\ 0,12 \end{matrix}$ 
↗
 $\begin{matrix} 12,08 \\ 0,04 \end{matrix}$ 
↗
 $\begin{matrix} 15,88 \\ 0,10 \end{matrix}$ 
↗
 $\begin{matrix} 19,55 \\ 0,07 \end{matrix}$ 
↗
 $\begin{matrix} 23,13 \\ 0,08 \end{matrix}$ 
↗
 $\begin{matrix} 26,83 \\ 0,18 \end{matrix}$

= MEDIA(xxx:yyy)  
 = DEV.ST.C(xxx:yyy)



# SEMIDIFFERENZE SALITA/DISCESA: CONTRIBUTO DELL'ATTRITO (DIPENDE SOLO DALLA MASSA DEL VOLANO)

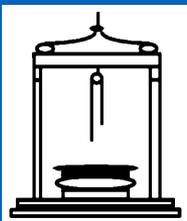
| N=0   |      | N=1   |      | N=2   |      | N=3   |      | N=4   |      | N=5   |      |
|-------|------|-------|------|-------|------|-------|------|-------|------|-------|------|
| 8,86  | 0,44 | 12,6  | 0,45 | -16,3 | 0,45 | -20   | 0,45 | -23,6 | 0,45 | 27,5  | 0,50 |
| 7,99  |      | 11,7  |      | -15,4 |      | -19,1 |      | -22,7 |      | 26,5  |      |
| -8,74 | 0,38 | -12,5 | 0,45 | 16,3  | 0,45 | 20,1  | 0,45 | 23,6  | 0,45 | -27   | 0,40 |
| -7,98 |      | -11,6 |      | 15,4  |      | 19,2  |      | 22,7  |      | -26,2 |      |
| 8,97  | 0,51 | 12,6  | 0,50 | -16,4 | 0,45 | -20   | 0,45 | -23,6 | 0,45 | 27,5  | 0,50 |
| 7,96  |      | 11,6  |      | -15,5 |      | -19,1 |      | -22,7 |      | 26,5  |      |
| -8,60 | 0,39 | -12,5 | 0,45 | 16,5  | 0,50 | 20    | 0,45 | 23,6  | 0,40 | -27,2 | 0,50 |
| -7,83 |      | -11,6 |      | 15,5  |      | 19,1  |      | 22,8  |      | -26,2 |      |
| 8,58  | 0,39 | 12,5  | 0,45 | -16,2 | 0,45 | -19,9 | 0,45 | -23,4 | 0,40 | 27,2  | 0,35 |
| 7,80  |      | 11,6  |      | -15,3 |      | -19   |      | -22,6 |      | 26,5  |      |
|       | 0,42 |       | 0,46 |       | 0,46 |       | 0,45 |       | 0,43 |       | 0,45 |

PER VALUTARE SE L'APPROSSIMAZIONE  
E' VALIDA ( $\alpha_{\text{attrito}}$  E' INDIPENDENTE DA N?)  
**VERIFICARE** (LabCalc) SE  $|p| < \sigma_p$

# LABORATORIO DI FISICA SPERIMENTALE

Ingegneria meccanica

A.A. 2023-2024



a giovedì 30 MAGGIO

lasciate il tavolo di laboratorio in ordine e pulito;  
ne siete responsabili (anche della strumentazione)

