1. Show that

$$xy \leq \frac{x^2}{2} + \frac{y^2}{2}, \qquad \text{for all}\, x, y \in \mathbb{R}$$

2. Show that

$$2xy \leq \epsilon x^2 + \frac{y^2}{\epsilon}, \qquad \text{for all}\, x, y \in \mathbb{R}, \epsilon > 0$$

3. Show that

$$(x+y)^2 \leq \frac{x^2}{t} + \frac{y^2}{1-t}, \qquad \text{for all}\, x, y \in \mathbb{R}, 0 < t < 1$$

4. Give the definition of a convex function in convex set C of $\mathbb{R}^n$.
5. Using **3.** show that x^2 is a convex function in $\mathbb{R}$.
6. Let $f^\star$ the conjugate of f, i.e. the Legendre trasform of f, defined by

$$f^\star(\zeta) = \sup_{x \in R^N} \{x\zeta - f(x)\} \qquad \zeta \in R^N$$

Show that if $f(x) = \frac{1}{p}|x|^p$ then $f^\star(x) = \frac{1}{q}|x|^q$, where $\frac{1}{p} + \frac{1}{q} = 1.$.