Exercize n.1 Given the functions $g_1(x) = x - 3$, $g_2(x) = -(x+2)^3$ find th set $\{x \in R : \, g_1(x) \leq 0 \, \text{and} \, g_2(x) \leq 0\}$.
Compute

$$g_1^+(x) = \max\{0, g_1(x)\}$$

and

$$g_2^+(x) = \max\{0, g_2(x)\}.$$

Study the regularity in $x = 0$.
Compute

$$g_1^+(x)^2, \; g_2^+(x)^2.$$

Study the regularity in $x = 0$. Find $F(x) = g_1^+(x) + g_2^+(x)$

A is an (n $\times$ n) symmetric matrix describing the coefficients of the quadratic terms.
$a \in \mathbb{R}^n \quad a \leq 0$ means $a_i \leq 0 \; \forall i = 1, \ldots, n$

Exercize n.2 Show the Cauchy-Shwarz inequality for the quadratic form

$$|A\lambda \cdot \mu| \leq (\sqrt{A\lambda \cdot \lambda})(\sqrt{A\mu \cdot \mu}),$$

for any $\lambda, \mu \in \mathbb{R}^n$
where A is a symmetric, positive semidefinite matrix.
Hint: $A\lambda \cdot \mu = A\mu \cdot \lambda$ for any $\lambda, \mu \in \mathbb{R}^n$.
Exercize n.3. Let A a symmetric matrix. Consider the quadratic form $Ax \cdot x$.
Compute the gradient of $F(x) = Ax \cdot x, \qquad x \in \mathbb{R}^n$,
Compute the Hessian matrix of $F(x) = Ax \cdot x, \qquad x \in \mathbb{R}^n$.
Exercize n.4. Quadratic Programming. Let A is a symmetric, positive definite matrix, $x \in \mathbb{R}^n$, $c \in \mathbb{R}^n$. Write the Karush-Kuhn-Tucker conditions for the QP minimization problem.

$$\min \frac{1}{2} Ax \cdot x + cx,$$

under the constraint $Qx \leq b, \quad b \in \mathbb{R}^m$ Q is an (m $\times$ n) matrix.
Exercize n.5. Write the Karush-Kuhn-Tucker conditions for the minimization problem

$$\min f(x) \qquad x \geq 0,$$

where $f : \mathbb{R}^n \to \mathbb{R}$ f is a differentiable function.