Risulta

$$\frac{\binom{2n+2}{n+1}}{\binom{2n}{n}} = \frac{(2n+2)(2n+1)2n\ldots(n+2)}{(n+1)!}\cdot\frac{n!}{2n(2n-1)\ldots(n+1)} =$$

$$= \frac{(2n+2)(2n+1)}{(n+1)^2}$$

e dunque

$$\lim_{n\to\infty}\frac{\binom{2n+2}{n+1}}{\binom{2n}{n}} = 4.$$

Per il teorema precedente, si ha allora anche

$$\lim_{n\to\infty}\sqrt[n]{\binom{2n}{n}} = 4.\ \blacksquare$$

Esercizi

Calcolare i seguenti limiti:

F 21. $\lim_{n\to\infty}\dfrac{\sqrt{n^3+9n^2}-\sqrt{n^4+1}}{n^2+2}$

F 22. $\lim_{n\to\infty}\dfrac{\sqrt{n^4+9n}-\sqrt{n^4+1}}{n^2+2n}$

F 23. $\lim_{n\to\infty}\sqrt[n]{n^4+1}$

F 24. $\lim_{n\to\infty}\sqrt[n]{2n^5+1}$

F 25. $\lim_{n\to\infty}\dfrac{n}{2^n-3^n}$

F 26. $\lim_{n\to\infty}\dfrac{n^2}{n!}$

F 27. $\lim_{n\to\infty}\dfrac{n^2+1}{n!}$

F 28. $\lim_{n\to\infty}n\left(\sqrt{n+1}-n\sqrt{n-1}\right)\sqrt{n^3+1}$

F 29. $\lim_{n\to\infty}\dfrac{n^{20}+4n^4+1}{2^n+5^n}$

F 30. $\lim_{n\to\infty}\dfrac{(n+1)^6-(n-1)^6}{(n+1)^5+(n-1)^5}$

F 31. $\lim_{n\to\infty}\sqrt[n]{2^n+3^n}$

F 32. $\lim_{n\to\infty}\left(\sqrt{1+\dfrac{2}{n^2}}-\sqrt{1-\dfrac{4}{n}}\right)$

F 33. $\lim_{n\to\infty}n\left(\sqrt{n^2+n}-n\right)$

M 34. $\lim_{n\to\infty}\left(1+\dfrac{1}{n!}\right)^{n}$

Limiti di successioni

M 35. $\lim_{n\to\infty}\sqrt[n]{a^{2n}+1}$ $(a\in\mathbb{R})$

M 36. $\lim_{n\to\infty}\dfrac{\ln(1+n+n^3)-3\ln n}{n\left(1-\cos\frac{1}{n^2}\right)}$

F 37. $\lim_{n\to\infty}\left(\dfrac{n}{n+1}-\dfrac{n+1}{n}\right)$

F 38. $\lim_{n\to\infty}\left(\dfrac{n^2}{n^2+1}-\dfrac{n^2}{n^2}\right)$

F 39. $\lim_{n\to\infty}\dfrac{\sin n}{n}$

F 40. $\lim_{n\to\infty}\dfrac{n!}{n^n}$

F 41. $\lim_{n\to\infty}(-1)^n\dfrac{n!}{n^n}$

M 42. $\lim_{n\to\infty}\dfrac{2n}{\sqrt[n]{n}}$

F 43. $\lim_{n\to\infty}\dfrac{\sqrt[n]{n!}}{n}$

F 44. $\lim_{n\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n}\right)$

D 45*. $\lim_{n\to\infty}\dfrac{1+\sqrt{2}+\sqrt[3]{3}+\ldots+\sqrt[n]{n}}{n}$

D 46. $\lim_{n\to\infty}\dfrac{\sqrt[n]{n(n+1)\ldots(2n)}}{n}$

F 47. $\lim_{n\to\infty}\dfrac{\ln n!}{n\ln n}$

D 48. $\lim_{n\to\infty}\sqrt[n]{n}\cdot\sqrt[n]{n+1}\ldots\sqrt[2n]{2n}$

F 49. $\lim_{n\to\infty}(n-\sqrt{n})\left(\sqrt[3]{1+\dfrac{2}{n}}-1\right)$

F 50. $\lim_{n\to\infty}\dfrac{\ln n^3}{\ln(n^3+3n^2)}$

F 51. $\lim_{n\to\infty}\sqrt[3]{8+\sin 2^{1/n}}-2$

M-D 52. $\lim_{n\to\infty}\left\{\left(\sqrt{n}\right)^n-3^n\right\}$

F 53. $\lim_{n\to\infty}\sqrt[n]{\dfrac{3n+2}{n^2}}$

F 54. $\lim_{n\to\infty}\dfrac{n^2+n\sin n}{1+n^2+n}$

F 55. $\lim_{n\to\infty}\sqrt[n]{\dfrac{n+2}{n}}$

M 56. $\lim_{n\to\infty}\dfrac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[4]{n^5-1}-\sqrt{n}}$

M-D 57. $\lim_{n\to\infty}\left(n^{\sqrt{n}}-2^n\right)$

F 58. $\lim_{n\to\infty}\dfrac{\sqrt{n^2+1}+\sqrt{n}}{\sqrt[4]{n^5-1}-\sqrt{n}}$

F 59. $\lim_{n\to\infty}\dfrac{7^n(1-n)}{1+n^2}$

F 60. $\lim_{n\to\infty}\dfrac{n!}{3^n}$

F 61. $\lim_{n\to\infty}[1+(-1)^n]$

F 62. $\lim_{n\to\infty}\left(\sqrt{n}-\sqrt[3]{n^3-n+\sin n}\right)$

F 63. $\lim_{n\to\infty}\dfrac{4n-3\times 2^n}{(5\times 2^{n-1}-3)\left(\sqrt{2n}-\sqrt{n-1}\right)}$

F 64. $\lim_{n\to\infty}\left(\dfrac{n^3}{n^2+2}-\dfrac{n^3-1}{n+1}\right)$

F 65. $\lim_{n\to\infty}\left(\dfrac{n^4+3}{n}-\dfrac{n^4+3n}{n-2}\right)$

F 66. $\lim_{n\to\infty}\left(\dfrac{n^3-2}{n^2}-\dfrac{n^3-2n^2}{n+1}\right)$

F 67. $\lim_{n\to\infty}\left(1+\dfrac{1}{n}\right)^{2n}$

F 68. $\lim_{n\to\infty}n^2\,2^{-\sqrt{n}}$