Proximity Methods in the Analysis of Subdivision Processes

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Abstract

Starting with [5], proximity methods have been employed in the analysis of nonlinear subdivision processes which are *analogous* to linear ones and which are still close enough to the latter so as not to lose important properties. This theory applies to geometric procedures which operate in Riemannian manifolds and other nonlinear geometries.

Topics investigated so far include smoothness for regular and irregular combinatorics [3, 6], approximation order, aspects of multiresolution analysis [1, 2], and stability [1] (these references are incomplete and in particalar omit work by G. Xie and T. Yu).

It appears to be possible, at least as far as the classes of *analogous constructions* are concerned, to reproduce the results from the linear theory, albeit with great technical difficulties in some places. It is interesting to study the different kinds of proximity inequalities needed, and their proofs: For instance, inferring the stability of a nonlinear scheme T from stability of a linear scheme S requires a *differential* proximity condition of the type

$$\|dT_p - S\| \leq \text{const} \cdot (\|\Delta p\|_\infty)^\beta \quad (\beta > 0).$$

With a look to future developments, it is noteworthy to give thought to the interrelation of different kinds of properties which subdivision processes might enjoy (such as the recent result that stability implies approximation order [4]).

This presentation gives an overview of the current status of work.

References

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