Full rank interpolatory subdivision schemes: factorizations, filters and encounters with the multivariate world

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Abstract

A fundamental number associated to a stationary vector subdivision scheme is the so called *rank* of the subdivision scheme. To be more precise, let $\mathbf{A} : \mathbb{Z}^s \to \mathbb{R}^{r\times r}$ be the *mask* of the subdivision scheme, defining the *subdivision operator*

$$S_{\mathbf{A}}\mathbf{c} = \sum_{\alpha\in\mathbb{Z}^s} \mathbf{A}\left(\cdot - 2\alpha\right)\, \mathbf{c}(\alpha), \qquad \mathbf{c} : \mathbb{Z}^s \to \mathbb{R}^r,$$

then the *rank* of $\mathbf{A}$ is defined to be the dimension of the joint eigenspace of the matrices

$$\mathbf{A}_\epsilon = \sum_{\alpha\in\mathbb{Z}^s} \mathbf{A}\left(\epsilon + 2\alpha\right), \qquad \epsilon \in \mathbb{Z}^s.$$

Rank considerations are crucial, for example, when factorizing subdivision operators in order to characterize their convergence by means of spectral radii.

A subdivision scheme if said to be of *full rank* if it has the maximal rank r and it is called *interpolatory* provided that $S_{\mathbf{A}}\mathbf{c}(2\cdot) = \mathbf{c}$, i.e., if it preserves the values obtained at earlier iterations. Full rank interpolatory schemes are natural in the sense that they are the ones which preserve constant *vector valued data*. But in fact more is true. In the univariate case such schemes can be used for the construction of matrix wavelets, they can be used to set up various types of filterbanks for handling vector data (an interesting application is, for example, the analysis of EEG data) and some of the results can also be translated to the multivariate case.

The talk will review some basic and not so basic aspects of full rank interpolatory subdivision schemes (univariate and multivariate) as well as present some open questions.

References

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