Refinable functions and shape preserving properties

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Abstract

It is well known the role played by total positivity in many fields and, in particular, in constructing approximations of a given data set with optimal properties [4]. For instance, in CAGD totally positive bases allow one to construct curves or surfaces which mimic the shape of a given set of control points, while in signal or image analysis they allow one to construct representations which preserve the sign and the monotonicity of the sampled data [9].

The construction of one dimensional refinable functions that are totally positive and, for that reason, enjoy shape preserving properties, has been dealt in [5], in case of dilation 2, and [6], in case of general dilation. In those papers, however, none explicit form of such masks has been provided except for the case of the binomial masks, i.e. the refinement masks associated to the B-splines on integer knots.

The problem of identifying explicit expressions of masks generating totally positive bases has been dealt in [8], where certain stationary refinement masks giving rise to refinable totally positive bases were introduced. In this talk we will analyze in details their shape preserving properties from the point of view both of refinability and of subdivision schemes and will present a few refinable operators havig optimal approximation properties.

Moreover we will put in evidence that the masks introduced in [8] give rise to a variety of subdivision schemes: nonstationary schemes with both high regularity and good localization properties [2, 7]; interpolatory schemes [1]; multivariate stationary and nonstationary schemes [3, 10].

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