Soluzioni

(1) Gauss su S cilindrica:

$$E \cdot 2\pi r h = \int_0^r \frac{Q(r')}{\varepsilon_0} 2\pi r' h\, dr'$$

$$\rightarrow \vec{E} = \frac{\hat{r}}{\varepsilon_0 r} \int_0^r \rho\, r'\, dr'$$

$r<a$: $E=0$

$a<r<b$: $E = \frac{1}{\varepsilon_0 r} \int_a^r k r'^2 dr' = \frac{k}{3\varepsilon_0 r}(r^3-a^3)$

$r>b$: $E = \frac{1}{\varepsilon_0 r} \int_a^b k r'^2 dr' = \frac{k}{3\varepsilon_0 r}(b^3-a^3)$

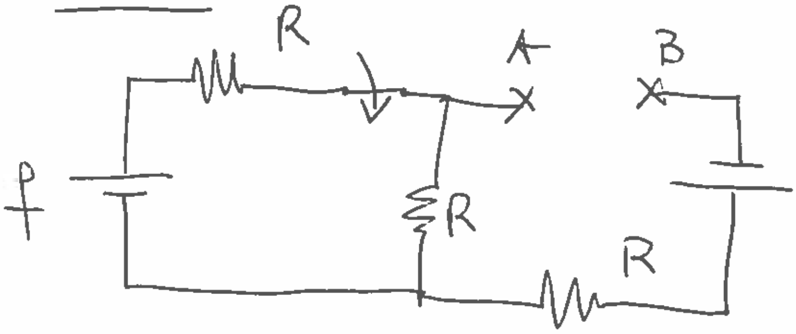
(2) Sia $dS = d\,dr$ elemento infinitesimo di superficie della sezione quadrata del solenoide a distanza $r \in [a, a+d]$

$$\Phi = \int_{\text{sez.}} B\, dS = N \int_a^{a+d} \frac{\mu_0 I}{2\pi r} d\, dr = \frac{N\mu_0 d}{2\pi} \ln\left(\frac{a+d}{a}\right) I$$

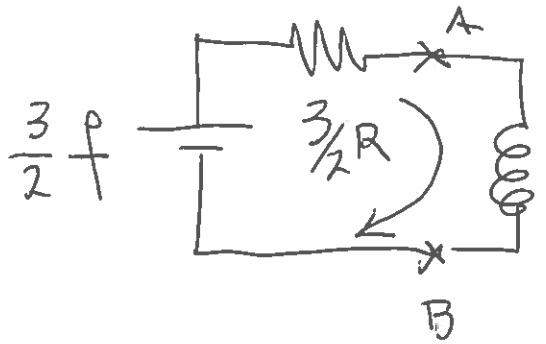
$$\Phi = M I \quad \Rightarrow \quad M = \frac{100 \cdot 4\pi 10^{-7}\, 2 \cdot 10^{-2}}{2\pi} \ln 3 \simeq 0.4\,\mu H$$

③ $\underline{t>0}$

$I_L(0) = \frac{f}{2R}$



$$\begin{cases} f_{eq} = \Delta V_{AB} = R\frac{f}{2R} + f = \frac{3}{2}f \\ R_{eq} = \frac{R}{2} + R = \frac{3}{2}R \end{cases}$$



$$\frac{3}{2}f - L\frac{dI}{dt} = \frac{3}{2}RI$$

$$\frac{f}{R} - \frac{2L/3R}{dt}dI = I \Rightarrow \frac{d(I - f/R)}{I - f/R} = -\frac{dt}{2L/3R}$$

$$I(t) - f/R = \left[I(0) - f/R\right]e^{-t/\tau} \quad \text{con } \tau = \frac{2L}{3R}$$

$$I(t) = \frac{f}{R} + \left(\frac{f}{2R} - \frac{f}{R}\right)e^{-t/\tau} = \frac{f}{R}\left(1 - \frac{1}{2}e^{-t/\tau}\right)$$

(4)

$$\vec{F}_L = q\vec{v} \times \vec{B} = q\omega r B \hat{r} \; ; \; 0 < r < \ell \; ;$$

$\hat{r}$ versore uscente da A

$$\vec{E}_S = -\frac{\vec{F}_L}{q} \rightarrow \text{equilibrio statico.}$$

$$\Delta V_{AC} = V_A - V_C = -\int_C^A \vec{E}_S \cdot d\vec{l} = \int_\ell^0 \omega r B \, dr$$

$$\Delta V_{AC} = -\frac{\omega B \ell^2}{2}$$

(5) $$\lambda = \frac{2\pi}{K} = \frac{2\pi}{3} \simeq 2m = 20a \gg a$$

$$f_i = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left(B\pi e^2\right) = -\pi e^2 \frac{d}{dt}\left[B_0 \cos(\omega t)\right]$$

$$f_i = \pi a^2 \frac{E_0}{c} \omega \sin\omega t = \pi a^2 \sqrt{2 Z_0 I}\, K \sin\omega t$$

$$i_{MAX} = \frac{\pi e^2 \sqrt{2 Z_0 I}}{R} K$$