

Complex structures on \mathbb{R}^6

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Dedicated to 60th birthday of Sylvestre Gallot

After a brief survey of the theory of orthogonal complex structures on even-dimensional Euclidean spaces, we present a classification result in the case of real dimension 6 that contrasts with the 4-dimensional picture. We prove that any ‘finite-energy’ complex structure has the form $J_0 \oplus J_1(z)$ relative to a splitting $\mathbb{R}^6 = \mathbb{C} \oplus \mathbb{R}^4$, where $J_1(z)$ is a linear structure on \mathbb{R}^4 that depends only on $z \in \mathbb{C}$. The main object of study is the twistor space of the sphere S^6 , which is well known to be the complex quadric $Q^6 \subset \mathbb{C}\mathbb{P}^7$, and an associated action of $SO(7, 1)$. Triality and various linear subspaces of Q^6 play a key role in the classification. The work is joint with L. Borisov and J. Viaclovsky.