Iteration of quadrilateral foldings

Starting with a quadrilateral $q_0 = (A_1, A_2, A_3, A_4)$ in the euclidean plane, one constructs a sequence of quadrilaterals $q_n = (A_{4n+1}, \ldots, A_{4n+4})$ by iteration of foldings, that is $q_n = \phi_4 \circ \phi_3 \circ \phi_2 \circ \phi_1 (q_{n-1})$, where the folding operation ϕ_j replaces the vertex number j by its symmetric with respect to the opposite diagonal.

We study the dynamical behavior of this sequence. In particular, we prove that :

- The drift $v := \lim\limits_{n\to\infty} \frac{1}{n} q_n$ exists.
- When none of the q_n is isometric to q_0, then the drift v is zero if and only if one has $\max a_j + \min a_j \leq \frac{1}{2} \sum a_j$, where $a_1, \ldots, a_4$ are the sidelengths of q_0.
- For Lebesgue almost all q_0 the normalized sequence $(q_n - nv)_{n\geq 1}$ is dense on a bounded analytic curve. However, for Baire generic q_0, the sequence $(q_n - nv)_{n\geq 1}$ is unbounded.