Growth of nonuniform lattices in negative curvature

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jointed work with M.Peigné, J-C Picaud and A.Sambusetti

Dedicated to 60th birthday of Sylvestre Gallot

Let X be a complete and simply connected Riemannian manifold with bounded negative sectional curvature. By definition the volume entropy of X is

$$W(X) = \limsup_{R \to +\infty} \frac{\ln V(x,R)}{R},$$

where $V(x,R)$ is the volume of the ball centered at x with radius R.

By analogy, the growth of a discrete group G acting on X by isometries is

$$\delta(G) = \limsup_{R \to +\infty} \frac{\ln V(x,R,G)}{R},$$

where $V(x,R,G)$ is the number of $g \in G$ such that $d(x, g(x)) < R$.

For any group G, we have $\delta(G) \leq W(X)$.

When the manifold G/X is compact, $W(X)$ and $\delta(G)$ are equal.

What happens when G/X has finite volume but is not compact (i-e G is a nonuniform lattice)?

The answer depends on the sectional curvature of X. Moreprecisely we proved the following theorems (*to appear in Journal fur die reine und angewandte Mathematik*):

Theorem 1 *If the sectional curvature of X is $\frac{1}{4}$- pinched, then for any nonuniform lattice G, we have $W(X) = \delta(G)$.*

Theorem 2 *There exists a complete and simply connected Riemannian surface X with bounded negative sectional curvature admitting a nonuniform lattice G satistying*

$$\delta(G) < W(X)$$

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The goal of my talk is to give the outline of the proof of Theorem 2.